PUZZLER

At sunset, the sky is ablaze with brilliant reds, pinks, and oranges. Yet, we wouldn't be able to see this sunset were it not for the fact that someone else is simultaneously seeing a blue sky. What causes the beautiful colors of a sunset, and why must the sky be blue somewhere else for us to enjoy one? *(© W. A. Banaszewski/Visuals Unlimited)*

chapter 38

Diffraction and Polarization

Chapter Outline

38.1 Introduction to Diffraction
38.2 Diffraction from Narrow Slits
38.3 Resolution of Single-Slit and Circular Apertures
38.4 The Diffraction Grating
38.5 *(Optional)* Diffraction of X-Rays by Crystals
38.6 Polarization of Light Waves

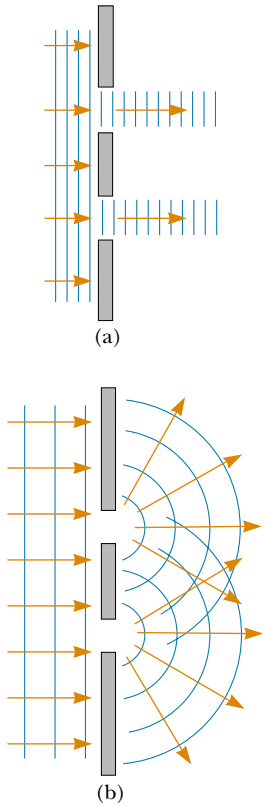


Figure 38.1 (a) If light waves did not spread out after passing through the slits, no interference would occur. (b) The light waves from the two slits overlap as they spread out, filling what we expect to be shadowed regions with light and producing interference fringes.

When light waves pass through a small aperture, an interference pattern is observed rather than a sharp spot of light. This behavior indicates that light, once it has passed through the aperture, spreads beyond the narrow path defined by the aperture into regions that would be in shadow if light traveled in straight lines. Other waves, such as sound waves and water waves, also have this property of spreading when passing through apertures or by sharp edges. This phenomenon, known as diffraction, can be described only with a wave model for light.

In Chapter 34, we learned that electromagnetic waves are transverse. That is, the electric and magnetic field vectors are perpendicular to the direction of wave propagation. In this chapter, we see that under certain conditions these transverse waves can be polarized in various ways.

38.1 INTRODUCTION TO DIFFRACTION

In Section 37.2 we learned that an interference pattern is observed on a viewing screen when two slits are illuminated by a single-wavelength light source. If the light traveled only in its original direction after passing through the slits, as shown in Figure 38.1a, the waves would not overlap and no interference pattern would be seen. Instead, Huygens's principle requires that the waves spread out from the slits as shown in Figure 38.1b. In other words, the light deviates from a straight-line path and enters the region that would otherwise be shadowed. As noted in Section 35.1, this divergence of light from its initial line of travel is called **diffraction.**

In general, diffraction occurs when waves pass through small openings, around obstacles, or past sharp edges, as shown in Figure 38.2. When an opaque object is placed between a point source of light and a screen, no sharp boundary exists on the screen between a shadowed region and an illuminated region. The illuminated region above the shadow of the object contains alternating light and dark fringes. Such a display is called a **diffraction pattern.**

Figure 38.3 shows a diffraction pattern associated with the shadow of a penny. A bright spot occurs at the center, and circular fringes extend outward from the shadow's edge. We can explain the central bright spot only by using the wave the-

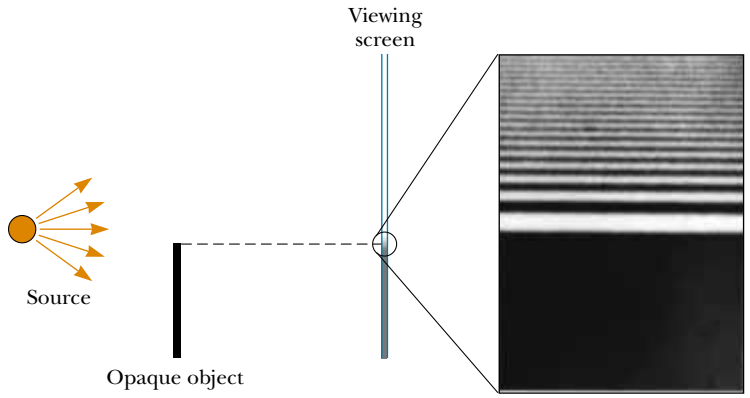


Figure 38.2 Light from a small source passes by the edge of an opaque object. We might expect no light to appear on the screen below the position of the edge of the object. In reality, light bends around the top edge of the object and enters this region. Because of these effects, a diffraction pattern consisting of bright and dark fringes appears in the region above the edge of the object.

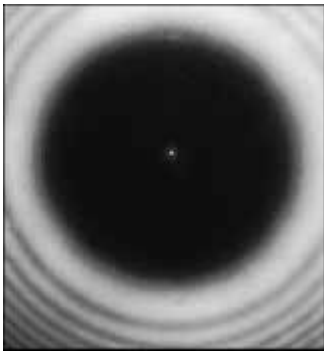


Figure 38.3 Diffraction pattern created by the illumination of a penny, with the penny positioned midway between screen and light source.

ory of light, which predicts constructive interference at this point. From the viewpoint of geometric optics (in which light is viewed as rays traveling in straight lines), we expect the center of the shadow to be dark because that part of the viewing screen is completely shielded by the penny.

It is interesting to point out an historical incident that occurred shortly before the central bright spot was first observed. One of the supporters of geometric optics, Simeon Poisson, argued that if Augustin Fresnel's wave theory of light were valid, then a central bright spot should be observed in the shadow of a circular object illuminated by a point source of light. To Poisson's astonishment, the spot was observed by Dominique Arago shortly thereafter. Thus, Poisson's prediction reinforced the wave theory rather than disproving it.

In this chapter we restrict our attention to **Fraunhofer diffraction,** which occurs, for example, when all the rays passing through a narrow slit are approximately parallel to one another. This can be achieved experimentally either by placing the screen far from the opening used to create the diffraction or by using a converging lens to focus the rays once they pass through the opening, as shown in Figure 38.4a. A bright fringe is observed along the axis at $\theta = 0$, with alternating dark and bright fringes occurring on either side of the central bright one. Figure 38.4b is a photograph of a single-slit Fraunhofer diffraction pattern.

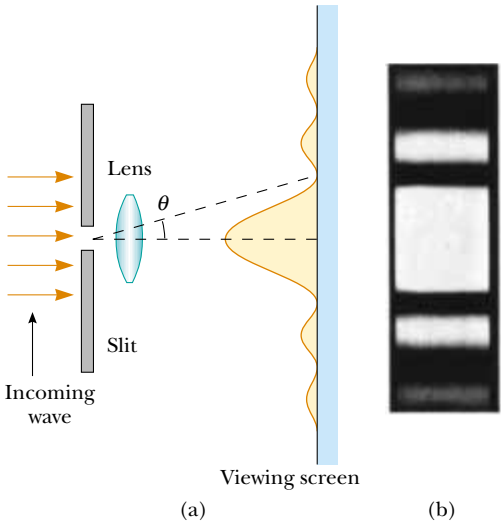


Figure 38.4 (a) Fraunhofer diffraction pattern of a single slit. The pattern consists of a central bright fringe flanked by much weaker maxima alternating with dark fringes (drawing not to scale). (b) Photograph of a single-slit Fraunhofer diffraction pattern.

38.2 DIFFRACTION FROM NARROW SLITS

Until now, we have assumed that slits are point sources of light. In this section, we abandon that assumption and see how the finite width of slits is the basis for understanding Fraunhofer diffraction.

We can deduce some important features of this phenomenon by examining waves coming from various portions of the slit, as shown in Figure 38.5. According to Huygens's principle, **each portion of the slit acts as a source of light waves.** Hence, light from one portion of the slit can interfere with light from another portion, and the resultant light intensity on a viewing screen depends on the direction θ.

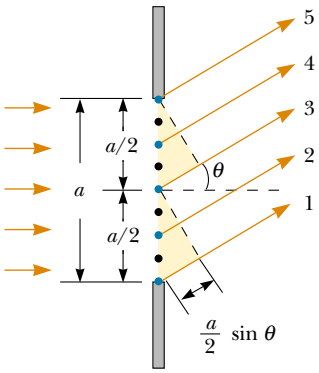


Figure 38.5 Diffraction of light by a narrow slit of width a. Each portion of the slit acts as a point source of light waves. The path difference between rays 1 and 3 or between rays 2 and 4 is $(a/2)\sin\theta$ (drawing not to scale).

To analyze the diffraction pattern, it is convenient to divide the slit into two halves, as shown in Figure 38.5. Keeping in mind that all the waves are in phase as they leave the slit, consider rays 1 and 3. As these two rays travel toward a viewing screen far to the right of the figure, ray 1 travels farther than ray 3 by an amount equal to the path difference $(a/2)\sin\theta$, where a is the width of the slit. Similarly, the path difference between rays 2 and 4 is also $(a/2)\sin\theta$. If this path difference is exactly half a wavelength (corresponding to a phase difference of 180°), then the two waves cancel each other and destructive interference results. This is true for any two rays that originate at points separated by half the slit width because the phase difference between two such points is 180°. Therefore, waves from the upper half of the slit interfere destructively with waves from the lower half when

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

or when

$$\sin\theta = \frac{\lambda}{a}$$

If we divide the slit into four equal parts and use similar reasoning, we find that the viewing screen is also dark when

$$\sin\theta = \frac{2\lambda}{a}$$

Likewise, we can divide the slit into six equal parts and show that darkness occurs on the screen when

$$\sin\theta = \frac{3\lambda}{a}$$

Therefore, the general condition for destructive interference is

Condition for destructive interference

$$\sin\theta = m\frac{\lambda}{a} \qquad m = \pm 1, \pm 2, \pm 3, \ldots \tag{38.1}$$

This equation gives the values of θ for which the diffraction pattern has zero light intensity—that is, when a dark fringe is formed. However, it tells us nothing about the variation in light intensity along the screen. The general features of the intensity distribution are shown in Figure 38.6. A broad central bright fringe is ob-

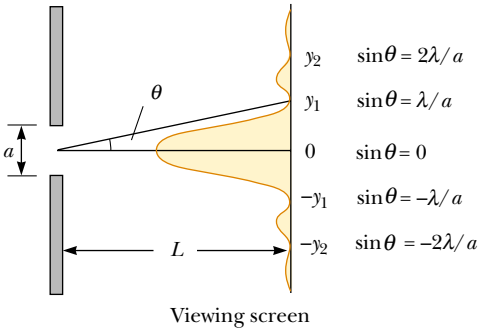
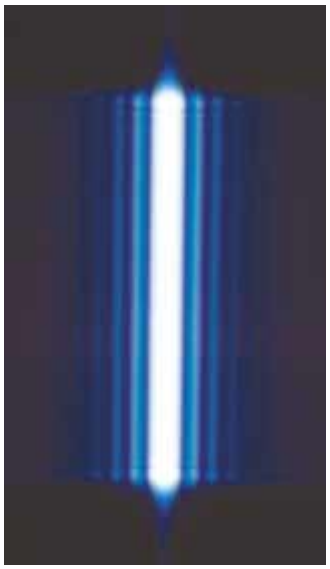


Figure 38.6 Intensity distribution for a Fraunhofer diffraction pattern from a single slit of width a. The positions of two minima on each side of the central maximum are labeled (drawing not to scale).

served; this fringe is flanked by much weaker bright fringes alternating with dark fringes. The various dark fringes occur at the values of θ that satisfy Equation 38.1. Each bright-fringe peak lies approximately halfway between its bordering dark-fringe minima. Note that the central bright maximum is twice as wide as the secondary maxima.



The diffraction pattern that appears on a screen when light passes through a narrow vertical slit. The pattern consists of a broad central bright fringe and a series of less intense and narrower side bright fringes.

Quick Quiz 38.1

If the door to an adjoining room is slightly ajar, why is it that you can hear sounds from the room but cannot see much of what is happening in the room?

Example 38.1 **Where Are the Dark Fringes?**

Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit. Find the positions of the first dark fringes and the width of the central bright fringe.

Solution The two dark fringes that flank the central bright fringe correspond to $m = \pm 1$ in Equation 38.1. Hence, we find that

$$\sin\theta = \pm\frac{\lambda}{a} = \pm\frac{5.80 \times 10^{-7}\ \text{m}}{0.300 \times 10^{-3}\ \text{m}} = \pm 1.93 \times 10^{-3}$$

From the triangle in Figure 38.6, note that $\tan\theta = y_1/L$. Because θ is very small, we can use the approximation $\sin\theta \approx \tan\theta$; thus, $\sin\theta \approx y_1/L$. Therefore, the positions of the first minima measured from the central axis are given by

$$y_1 \approx L\sin\theta = \pm L\frac{\lambda}{a} = \pm 3.87 \times 10^{-3}\ \text{m}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe. Hence, the width of the central bright fringe is equal to $2|y_1| = 7.74 \times 10^{-3}\ \text{m} = 7.74\ \text{mm}$. Note that this value is much greater than the width of the slit. However, as the slit width is increased, the diffraction pattern narrows, corresponding to smaller values of θ. In fact, for large values of a, the various maxima and minima are so closely spaced that only a large central bright area resembling the geometric image of the slit is observed. This is of great importance in the design of lenses used in telescopes, microscopes, and other optical instruments.

Exercise Determine the width of the first-order ($m = 1$) bright fringe.

Answer 3.87 mm.

Intensity of Single-Slit Diffraction Patterns

We can use phasors to determine the light intensity distribution for a single-slit diffraction pattern. Imagine a slit divided into a large number of small zones, each of width Δy as shown in Figure 38.7. Each zone acts as a source of coherent radiation,

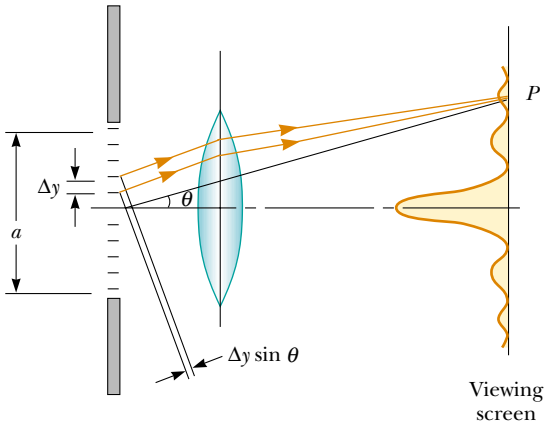


Figure 38.7 Fraunhofer diffraction by a single slit. The light intensity at point P is the resultant of all the incremental electric field magnitudes from zones of width Δy.

QuickLab

Make a V with your index and middle fingers. Hold your hand up very close to your eye so that you are looking between your two fingers toward a bright area. Now bring the fingers together until there is only a very tiny slit between them. You should be able to see a series of parallel lines. Although the lines appear to be located in the narrow space between your fingers, what you are actually seeing is a diffraction pattern cast upon your retina.

and each contributes an incremental electric field of magnitude ΔE at some point P on the screen. We obtain the total electric field magnitude E at point P by summing the contributions from all the zones. The light intensity at point P is proportional to the square of the magnitude of the electric field (see Section 37.3).

The incremental electric field magnitudes between adjacent zones are out of phase with one another by an amount $\Delta\beta$, where the phase difference $\Delta\beta$ is related to the path difference $\Delta y \sin\theta$ between adjacent zones by the expression

$$\Delta\beta = \frac{2\pi}{\lambda}\,\Delta y \sin\theta \qquad \textbf{(38.2)}$$

To find the magnitude of the total electric field on the screen at any angle θ, we sum the incremental magnitudes ΔE due to each zone. For small values of θ, we can assume that all the ΔE values are the same. It is convenient to use phasor diagrams for various angles, as shown in Figure 38.8. When $\theta = 0$, all phasors are aligned as shown in Figure 38.8a because all the waves from the various zones are in phase. In this case, the total electric field at the center of the screen is $E_0 = N\,\Delta E$, where N is the number of zones. The resultant magnitude E_R at some small angle θ is shown in Figure 38.8b, where each phasor differs in phase from an adjacent one by an amount $\Delta\beta$. In this case, E_R is the vector sum of the incremental

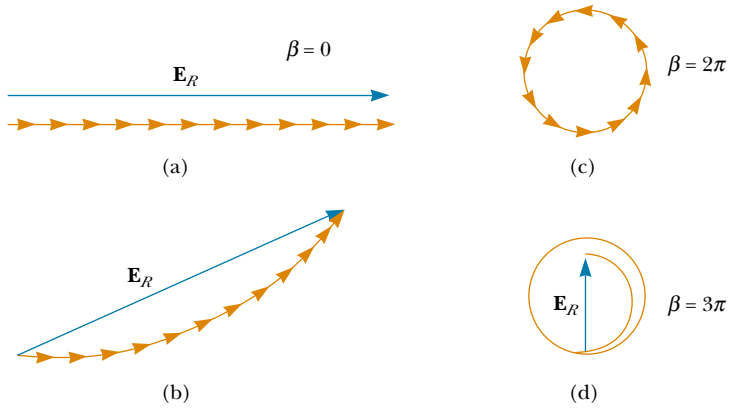


Figure 38.8 Phasor diagrams for obtaining the various maxima and minima of a single-slit diffraction pattern.

magnitudes and hence is given by the length of the chord. Therefore, $E_R < E_0$. The total phase difference β between waves from the top and bottom portions of the slit is

$$\beta = N\Delta\beta = \frac{2\pi}{\lambda} N\Delta y \sin\theta = \frac{2\pi}{\lambda} a \sin\theta \tag{38.3}$$

where $a = N\Delta y$ is the width of the slit.

As θ increases, the chain of phasors eventually forms the closed path shown in Figure 38.8c. At this point, the vector sum is zero, and so $E_R = 0$, corresponding to the first minimum on the screen. Noting that $\beta = N\Delta\beta = 2\pi$ in this situation, we see from Equation 38.3 that

$$2\pi = \frac{2\pi}{\lambda} a \sin\theta$$

$$\sin\theta = \frac{\lambda}{a}$$

That is, the first minimum in the diffraction pattern occurs where $\sin\theta = \lambda/a$; this is in agreement with Equation 38.1.

At greater values of θ, the spiral chain of phasors tightens. For example, Figure 38.8d represents the situation corresponding to the second maximum, which occurs when $\beta = 360° + 180° = 540°$ (3π rad). The second minimum (two complete circles, not shown) corresponds to $\beta = 720°$ (4π rad), which satisfies the condition $\sin\theta = 2\lambda/a$.

We can obtain the total electric field magnitude E_R and light intensity I at any point P on the screen in Figure 38.7 by considering the limiting case in which Δy becomes infinitesimal (dy) and N approaches ∞. In this limit, the phasor chains in Figure 38.8 become the red curve of Figure 38.9. The arc length of the curve is E_0 because it is the sum of the magnitudes of the phasors (which is the total electric field magnitude at the center of the screen). From this figure, we see that at some angle θ, the resultant electric field magnitude E_R on the screen is equal to the chord length. From the triangle containing the angle $\beta/2$, we see that

$$\sin\frac{\beta}{2} = \frac{E_R/2}{R}$$

where R is the radius of curvature. But the arc length E_0 is equal to the product $R\beta$, where β is measured in radians. Combining this information with the previous expression gives

$$E_R = 2R\sin\frac{\beta}{2} = 2\left(\frac{E_0}{\beta}\right)\sin\frac{\beta}{2} = E_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]$$

Because the resultant light intensity I at point P on the screen is proportional to the square of the magnitude E_R, we find that

$$I = I_{\max}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 \tag{38.4}$$

Intensity of a single-slit Fraunhofer diffraction pattern

where $I_{\max}$ is the intensity at $\theta = 0$ (the central maximum). Substituting the expression for β (Eq. 38.3) into Equation 38.4, we have

$$I = I_{\max}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2 \tag{38.5}$$

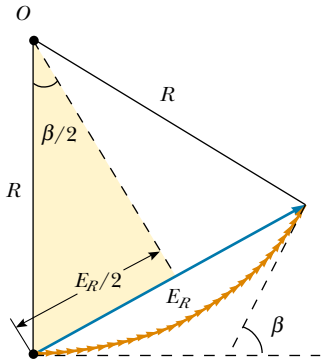


Figure 38.9 Phasor diagram for a large number of coherent sources. All the ends of the phasors lie on the circular red arc of radius R. The resultant electric field magnitude E_R equals the length of the chord.

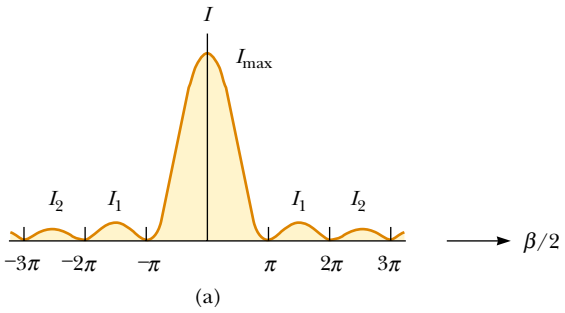


Figure 38.10 (a) A plot of light intensity I versus $\beta/2$ for the single-slit Fraunhofer diffraction pattern. (b) Photograph of a single-slit Fraunhofer diffraction pattern.

From this result, we see that minima occur when

$$\frac{\pi a \sin\theta}{\lambda} = m\pi$$

or

Condition for intensity minima

$$\sin\theta = m\frac{\lambda}{a} \qquad m = \pm 1, \pm 2, \pm 3, \ldots$$

in agreement with Equation 38.1.

Figure 38.10a represents a plot of Equation 38.5, and Figure 38.10b is a photograph of a single-slit Fraunhofer diffraction pattern. Note that most of the light intensity is concentrated in the central bright fringe.

EXAMPLE 38.2 Relative Intensities of the Maxima

Find the ratio of the intensities of the secondary maxima to the intensity of the central maximum for the single-slit Fraunhofer diffraction pattern.

Solution To a good approximation, the secondary maxima lie midway between the zero points. From Figure 38.10a, we see that this corresponds to $\beta/2$ values of $3\pi/2$, $5\pi/2$, $7\pi/2$, ... Substituting these values into Equation 38.4 gives for the first two ratios

$$\frac{I_1}{I_{max}} = \left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = \frac{1}{9\pi^2/4} = 0.045$$

$$\frac{I_2}{I_{max}} = \left[\frac{\sin(5\pi/2)}{5\pi/2}\right]^2 = \frac{1}{25\pi^2/4} = 0.016$$

That is, the first secondary maxima (the ones adjacent to the central maximum) have an intensity of 4.5% that of the central maximum, and the next secondary maxima have an intensity of 1.6% that of the central maximum.

Exercise Determine the intensity, relative to the central maximum, of the secondary maxima corresponding to $m = \pm 3$.

Answer 0.008 3.

Intensity of Two-Slit Diffraction Patterns

When more than one slit is present, we must consider not only diffraction due to the individual slits but also the interference of the waves coming from different slits. You may have noticed the curved dashed line in Figure 37.13, which indicates a decrease in intensity of the interference maxima as θ increases. This decrease is

due to diffraction. To determine the effects of both interference and diffraction, we simply combine Equation 37.12 and Equation 38.5:

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2 \qquad \textbf{(38.6)}$$

Although this formula looks complicated, it merely represents the diffraction pattern (the factor in brackets) acting as an "envelope" for a two-slit interference pattern (the cosine-squared factor), as shown in Figure 38.11.

Equation 37.2 indicates the conditions for interference maxima as $d \sin\theta = m\lambda$, where d is the distance between the two slits. Equation 38.1 specifies that the first diffraction minimum occurs when $a \sin\theta = \lambda$, where a is the slit width. Dividing Equation 37.2 by Equation 38.1 (with $m = 1$) allows us to determine which interference maximum coincides with the first diffraction minimum:

$$\frac{d \sin\theta}{a \sin\theta} = \frac{m\lambda}{\lambda}$$

$$\frac{d}{a} = m \qquad \textbf{(38.7)}$$

In Figure 38.11, $d/a = 18\ \mu\text{m}/3.0\ \mu\text{m} = 6$. Thus, the sixth interference maximum (if we count the central maximum as $m = 0$) is aligned with the first diffraction minimum and cannot be seen.

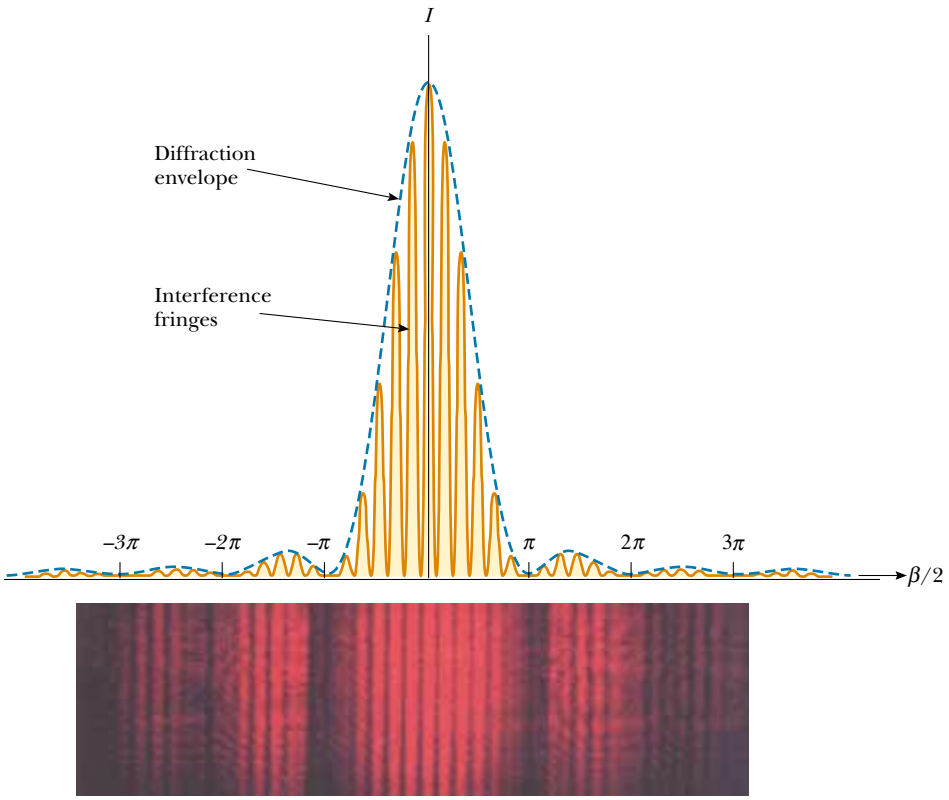


Figure 38.11 The combined effects of diffraction and interference. This is the pattern produced when 650-nm light waves pass through two 3.0-μm slits that are 18 μm apart. Notice how the diffraction pattern acts as an "envelope" and controls the intensity of the regularly spaced interference maxima.

Quick Quiz 38.2

Using Figure 38.11 as a starting point, make a sketch of the combined diffraction and interference pattern for 650-nm light waves striking two 3.0-μm slits located 9.0 μm apart.

38.3 RESOLUTION OF SINGLE-SLIT AND CIRCULAR APERTURES

The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light. To understand this difficulty, let us consider Figure 38.12, which shows two light sources far from a narrow slit of width a. The sources can be considered as two noncoherent point sources S_1 and S_2—for example, they could be two distant stars. If no diffraction occurred, two distinct bright spots (or images) would be observed on the viewing screen. However, because of diffraction, each source is imaged as a bright central region flanked by weaker bright and dark fringes. What is observed on the screen is the sum of two diffraction patterns: one from S_1, and the other from S_2.

If the two sources are far enough apart to keep their central maxima from overlapping, as shown in Figure 38.12a, their images can be distinguished and are said to be *resolved*. If the sources are close together, however, as shown in Figure 38.12b, the two central maxima overlap, and the images are not resolved. In determining whether two images are resolved, the following condition is often used:

When the central maximum of one image falls on the first minimum of the other image, the images are said to be just resolved. This limiting condition of resolution is known as **Rayleigh's criterion.**

Figure 38.13 shows diffraction patterns for three situations. When the objects are far apart, their images are well resolved (Fig. 38.13a). When the angular sepa-

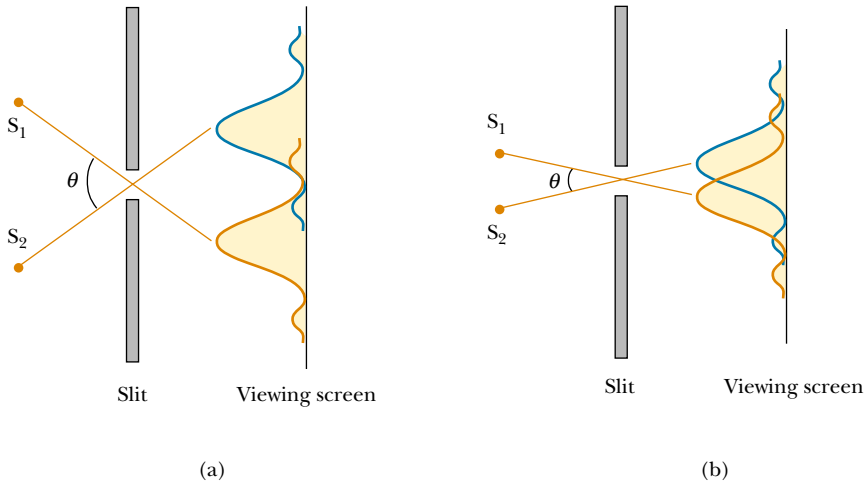


Figure 38.12 Two point sources far from a narrow slit each produce a diffraction pattern. (a) The angle subtended by the sources at the slit is large enough for the diffraction patterns to be distinguishable. (b) The angle subtended by the sources is so small that their diffraction patterns overlap, and the images are not well resolved. (Note that the angles are greatly exaggerated. The drawing is not to scale.)

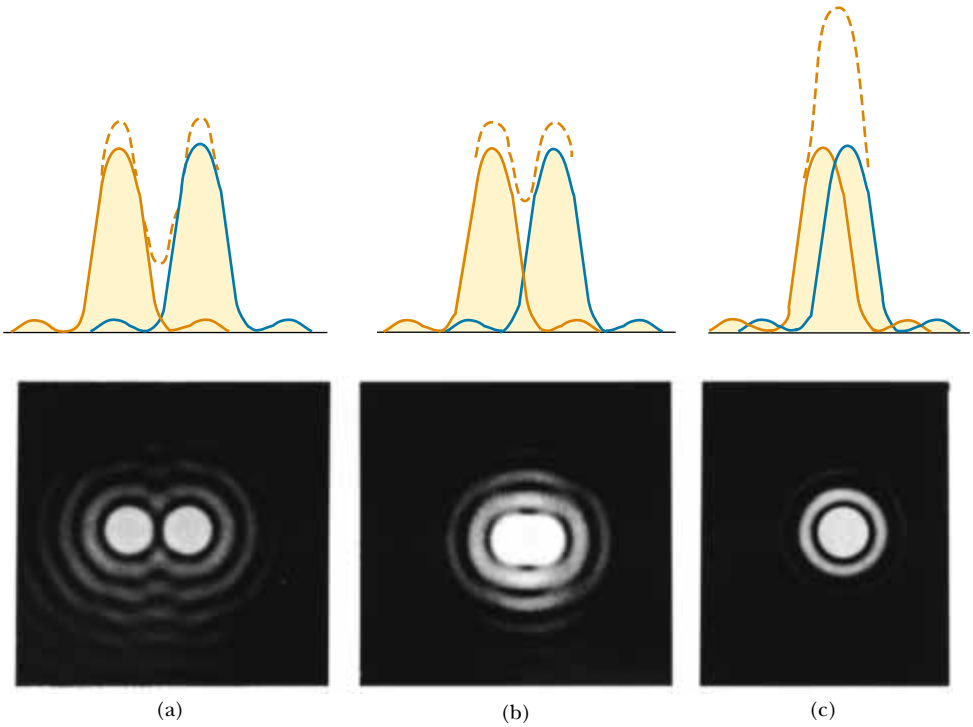


Figure 38.13 Individual diffraction patterns of two point sources (solid curves) and the resultant patterns (dashed curves) for various angular separations of the sources. In each case, the dashed curve is the sum of the two solid curves. (a) The sources are far apart, and the patterns are well resolved. (b) The sources are closer together such that the angular separation just satisfies Rayleigh's criterion, and the patterns are just resolved. (c) The sources are so close together that the patterns are not resolved.

ration of the objects satisfies Rayleigh's criterion (Fig. 38.13b), the images are just resolved. Finally, when the objects are close together, the images are not resolved (Fig. 38.13c).

From Rayleigh's criterion, we can determine the minimum angular separation θ_{min} subtended by the sources at the slit for which the images are just resolved. Equation 38.1 indicates that the first minimum in a single-slit diffraction pattern occurs at the angle for which

$$\sin \theta = \frac{\lambda}{a}$$

where a is the width of the slit. According to Rayleigh's criterion, this expression gives the smallest angular separation for which the two images are resolved. Because $\lambda \ll a$ in most situations, $\sin \theta$ is small, and we can use the approximation $\sin \theta \approx \theta$. Therefore, the limiting angle of resolution for a slit of width a is

$$\theta_{\text{min}} = \frac{\lambda}{a} \tag{38.8}$$

where θ_{min} is expressed in radians. Hence, the angle subtended by the two sources at the slit must be greater than λ / a if the images are to be resolved.

Many optical systems use circular apertures rather than slits. The diffraction pattern of a circular aperture, shown in Figure 38.14, consists of a central circular

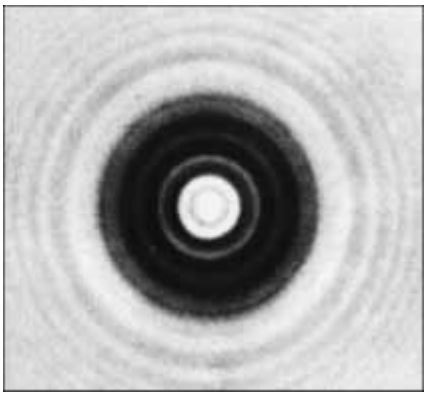


Figure 38.14 The diffraction pattern of a circular aperture consists of a central bright disk surrounded by concentric bright and dark rings.

bright disk surrounded by progressively fainter bright and dark rings. Analysis shows that the limiting angle of resolution of the circular aperture is

Limiting angle of resolution for a circular aperture

$$\theta_{\min} = 1.22 \frac{\lambda}{D} \tag{38.9}$$

where D is the diameter of the aperture. Note that this expression is similar to Equation 38.8 except for the factor 1.22, which arises from a complex mathematical analysis of diffraction from the circular aperture.

EXAMPLE 38.3 Limiting Resolution of a Microscope

Light of wavelength 589 nm is used to view an object under a microscope. If the aperture of the objective has a diameter of 0.900 cm, (a) what is the limiting angle of resolution?

Solution (a) Using Equation 38.9, we find that the limiting angle of resolution is

$$\theta_{\min} = 1.22\left(\frac{589 \times 10^{-9}\ \text{m}}{0.900 \times 10^{-2}\ \text{m}}\right) = 7.98 \times 10^{-5}\ \text{rad}$$

This means that any two points on the object subtending an angle smaller than this at the objective cannot be distinguished in the image.

(b) If it were possible to use visible light of any wavelength, what would be the maximum limit of resolution for this microscope?

Solution To obtain the smallest limiting angle, we have to use the shortest wavelength available in the visible spectrum. Violet light (400 nm) gives a limiting angle of resolution of

$$\theta_{\min} = 1.22\left(\frac{400 \times 10^{-9}\ \text{m}}{0.900 \times 10^{-2}\ \text{m}}\right) = 5.42 \times 10^{-5}\ \text{rad}$$

(c) Suppose that water ($n = 1.33$) fills the space between the object and the objective. What effect does this have on resolving power when 589-nm light is used?

Solution We find the wavelength of the 589-nm light in the water using Equation 35.7:

$$\lambda_{\text{water}} = \frac{\lambda_{\text{air}}}{n_{\text{water}}} = \frac{589\ \text{nm}}{1.33} = 443\ \text{nm}$$

The limiting angle of resolution at this wavelength is now smaller than that calculated in part (a):

$$\theta_{\min} = 1.22\left(\frac{443 \times 10^{-9}\ \text{m}}{0.900 \times 10^{-2}\ \text{m}}\right) = 6.00 \times 10^{-5}\ \text{rad}$$

EXAMPLE 38.4 Resolution of a Telescope

The Hale telescope at Mount Palomar has a diameter of 200 in. What is its limiting angle of resolution for 600-nm light?

Solution Because $D = 200\ \text{in.} = 5.08\ \text{m}$ and $\lambda = 6.00 \times 10^{-7}\ \text{m}$, Equation 38.9 gives

$$\theta_{\min} = 1.22 \frac{\lambda}{D} = 1.22\left(\frac{6.00 \times 10^{-7}\ \text{m}}{5.08\ \text{m}}\right)$$

$$= 1.44 \times 10^{-7}\ \text{rad} \approx 0.03\ \text{s of arc}$$

Any two stars that subtend an angle greater than or equal to this value are resolved (if atmospheric conditions are ideal).

The Hale telescope can never reach its diffraction limit because the limiting angle of resolution is always set by atmospheric blurring. This seeing limit is usually about 1 s of arc and is never smaller than about 0.1 s of arc. (This is one of the reasons for the superiority of photographs from the Hubble Space Telescope, which views celestial objects from an orbital position above the atmosphere.)

Exercise The large radio telescope at Arecibo, Puerto Rico, has a diameter of 305 m and is designed to detect 0.75-m radio waves. Calculate the minimum angle of resolution for this telescope and compare your answer with that for the Hale telescope.

Answer 3.0×10^{-3} rad (10 min of arc), more than 10 000 times larger (that is, *worse*) than the Hale minimum.

EXAMPLE 38.5 Resolution of the Eye

Estimate the limiting angle of resolution for the human eye, assuming its resolution is limited only by diffraction.

Solution Let us choose a wavelength of 500 nm, near the center of the visible spectrum. Although pupil diameter

varies from person to person, we estimate a diameter of 2 mm. We use Equation 38.9, taking $\lambda = 500$ nm and $D = 2$ mm:

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{5.00 \times 10^{-7}\ \text{m}}{2 \times 10^{-3}\ \text{m}}\right)$$

$$\approx \boxed{3 \times 10^{-4}\ \text{rad} \approx 1\ \text{min of arc}}$$

We can use this result to determine the minimum separation distance d between two point sources that the eye can distinguish if they are a distance L from the observer (Fig. 38.15). Because $\theta_{\min}$ is small, we see that

$$\sin\theta_{\min} \approx \theta_{\min} \approx \frac{d}{L}$$

$$d = L\theta_{\min}$$

For example, if the point sources are 25 cm from the eye (the near point), then

$$d = (25\ \text{cm})(3 \times 10^{-4}\ \text{rad}) = 8 \times 10^{-3}\ \text{cm}$$

This is approximately equal to the thickness of a human hair.

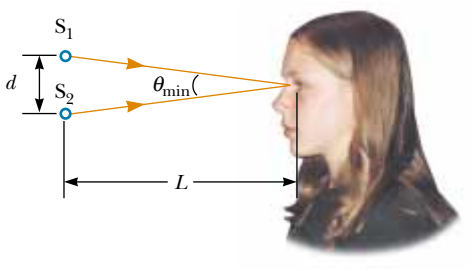


Figure 38.15 Two point sources separated by a distance d as observed by the eye.

Exercise Suppose that the pupil is dilated to a diameter of 5.0 mm and that two point sources 3.0 m away are being viewed. How far apart must the sources be if the eye is to resolve them?

Answer 0.037 cm.

APPLICATION Loudspeaker Design

The three-way speaker system shown in Figure 38.16 contains a woofer, a midrange speaker, and a tweeter. The small-diameter tweeter is for high frequencies, and the large-diameter woofer is for low frequencies. The midrange speaker, of intermediate diameter, is used for the frequency band above the high-frequency cutoff of the woofer and below the low-frequency cutoff of the tweeter. Circuits known as crossover networks include low-pass, midrange, and high-pass filters that direct the electrical signal to the appropriate speaker. The effective aperture size of a speaker is approximately its diameter. Because the wavelengths of sound waves are comparable to the typical sizes of the speakers, diffraction effects determine the angular radiation pattern. To be most useful, a speaker should radiate sound over a broad range of angles so that the listener does not have to stand at a particular spot in the room to hear maximum sound intensity. On the basis of the angular radiation pattern, let us investigate the frequency range for which a 6-in. (0.15-m) midrange speaker is most useful.

The speed of sound in air is 344 m/s, and for a circular aperture, diffraction effects become important when $\lambda = 1.22D$, where D is the speaker diameter. Therefore, we would expect this speaker to radiate non-uniformly for all frequencies above

$$\frac{344\ \text{m/s}}{1.22(0.15\ \text{m})} = 1\,900\ \text{Hz}$$

Suppose our design specifies that the midrange speaker operates between 500 Hz (the high-frequency woofer cutoff) and 2 000 Hz. Measurements of the dispersion of radiated

Figure 38.16 An audio speaker system for high-fidelity sound reproduction. The tweeter is at the top, the midrange speaker is in the middle, and the woofer is at the bottom. *(International Stock Photography)*

sound at a suitably great distance from the speaker yield the angular profiles of sound intensity shown in Figure 38.17. In examining these plots, we see that the dispersion pattern for a 500-Hz sound is fairly uniform. This angular range is sufficiently great for us to say that this midrange speaker satisfies the design criterion. The intensity of a 2 000-Hz sound decreases to about half its maximum value about 30° from the centerline.

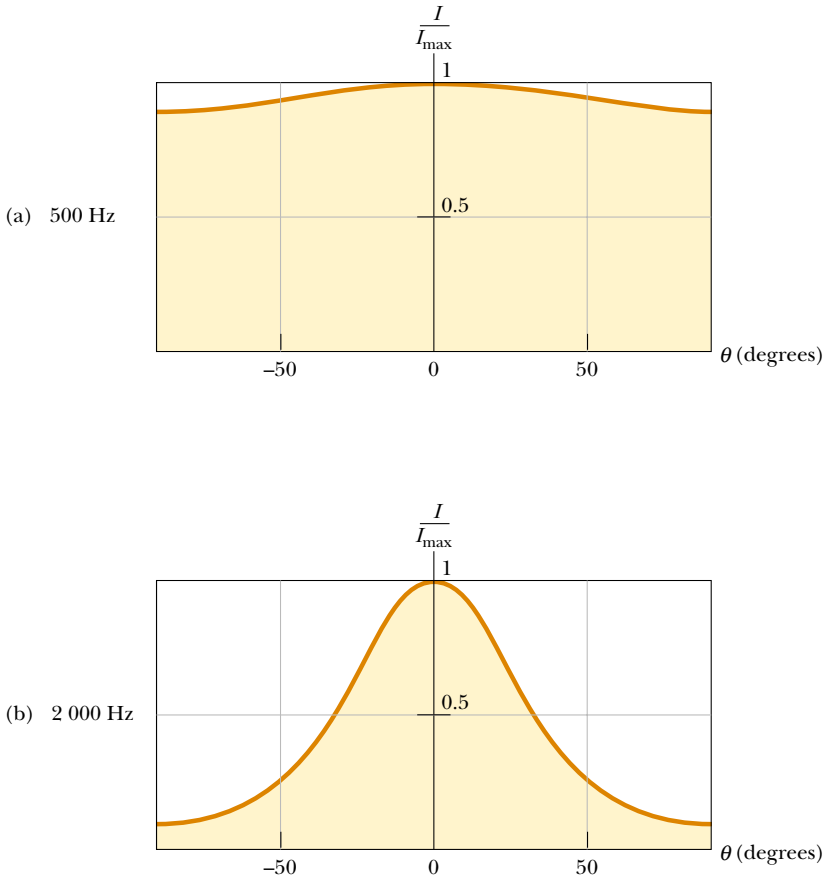


Figure 38.17 Angular dispersion of sound intensity I for a midrange speaker at (a) 500 Hz and (b) 2 000 Hz.

38.4 THE DIFFRACTION GRATING

The **diffraction grating,** a useful device for analyzing light sources, consists of a large number of equally spaced parallel slits. A *transmission grating* can be made by cutting parallel lines on a glass plate with a precision ruling machine. The spaces between the lines are transparent to the light and hence act as separate slits. A *reflection grating* can be made by cutting parallel lines on the surface of a reflective material. The reflection of light from the spaces between the lines is specular, and the reflection from the lines cut into the material is diffuse. Thus, the spaces between the lines act as parallel sources of reflected light, like the slits in a transmission grating. Gratings that have many lines very close to each other can have very small slit spacings. For example, a grating ruled with 5 000 lines/cm has a slit spacing $d = (1/5\,000)\ \text{cm} = 2.00 \times 10^{-4}\ \text{cm}$.

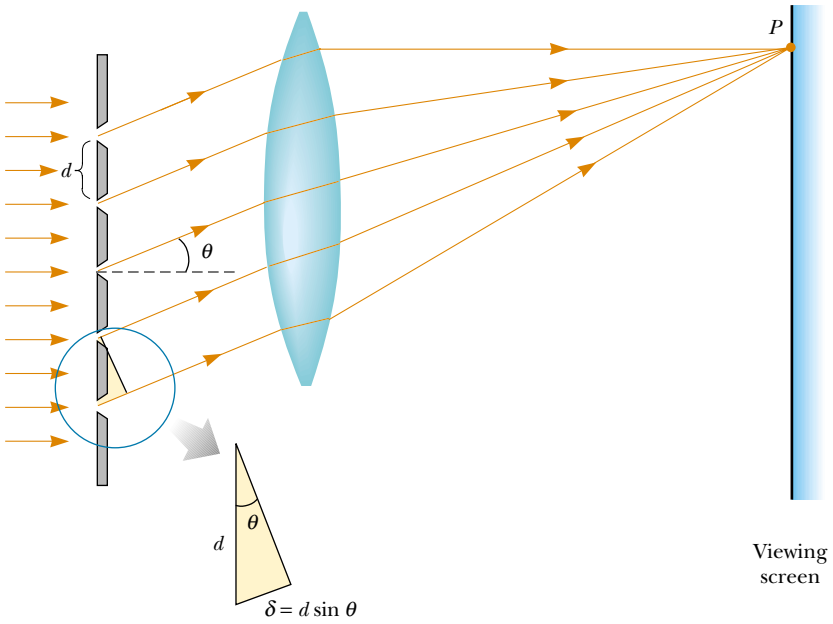


Figure 38.18 Side view of a diffraction grating. The slit separation is d, and the path difference between adjacent slits is $d \sin \theta$.

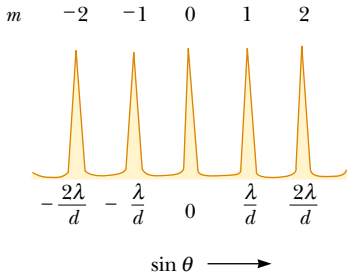


Figure 38.19 Intensity versus $\sin \theta$ for a diffraction grating. The zeroth-, first-, and second-order maxima are shown.

A section of a diffraction grating is illustrated in Figure 38.18. A plane wave is incident from the left, normal to the plane of the grating. A converging lens brings the rays together at point P. The pattern observed on the screen is the result of the combined effects of interference and diffraction. Each slit produces diffraction, and the diffracted beams interfere with one another to produce the final pattern.

The waves from all slits are in phase as they leave the slits. However, for some arbitrary direction θ measured from the horizontal, the waves must travel different path lengths before reaching point P. From Figure 38.18, note that the path difference δ between rays from any two adjacent slits is equal to $d \sin \theta$. If this path difference equals one wavelength or some integral multiple of a wavelength, then waves from all slits are in phase at point P and a bright fringe is observed. Therefore, the condition for maxima in the interference pattern at the angle θ is

$$d \sin \theta = m\lambda \qquad m = 0, 1, 2, 3, \ldots \qquad \textbf{(38.10)}$$

Condition for interference maxima for a grating

We can use this expression to calculate the wavelength if we know the grating spacing and the angle θ. If the incident radiation contains several wavelengths, the mth-order maximum for each wavelength occurs at a specific angle. All wavelengths are seen at $\theta = 0$, corresponding to $m = 0$, the zeroth-order maximum. The first-order maximum ($m = 1$) is observed at an angle that satisfies the relationship $\sin \theta = \lambda/d$; the second-order maximum ($m = 2$) is observed at a larger angle θ, and so on.

The intensity distribution for a diffraction grating obtained with the use of a monochromatic source is shown in Figure 38.19. Note the sharpness of the principal maxima and the broadness of the dark areas. This is in contrast to the broad bright fringes characteristic of the two-slit interference pattern (see Fig. 37.6). Because the principal maxima are so sharp, they are very much brighter than two-slit

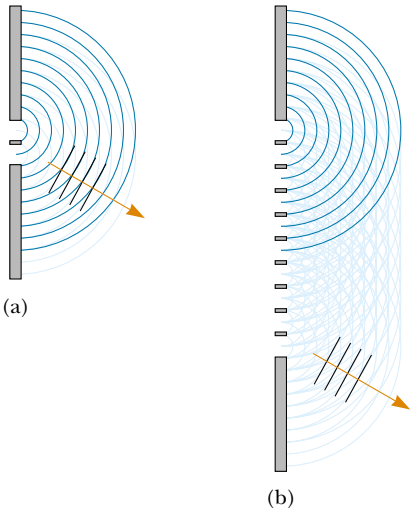


Figure 38.20 (a) Addition of two wave fronts from two slits. (b) Addition of ten wave fronts from ten slits. The resultant wave is much stronger in part (b) than in part (a).

QuickLab

Stand a couple of meters from a lightbulb. Facing away from the light, hold a compact disc about 10 cm from your eye and tilt it until the reflection of the bulb is located in the hole at the disc's center. You should see spectra radiating out from the center, with violet on the inside and red on the outside. Now move the disc away from your eye until the violet band is at the outer edge. Carefully measure the distance from your eye to the center of the disc and also determine the radius of the disc. Use this information to find the angle θ to the first-order maximum for violet light. Now use Equation 38.10 to determine the spacing between the grooves on the disc. The industry standard is 1.6 μm. How close did you come?

interference maxima. The reason for this is illustrated in Figure 38.20, in which the combination of multiple wave fronts for a ten-slit grating is compared with the wave fronts for a two-slit system. Actual gratings have thousands of times more slits, and therefore the maxima are even stronger.

A schematic drawing of a simple apparatus used to measure angles in a diffraction pattern is shown in Figure 38.21. This apparatus is a diffraction grating spectrometer. The light to be analyzed passes through a slit, and a collimated beam of light is incident on the grating. The diffracted light leaves the grating at angles that satisfy Equation 38.10, and a telescope is used to view the image of the slit. The wavelength can be determined by measuring the precise angles at which the images of the slit appear for the various orders.

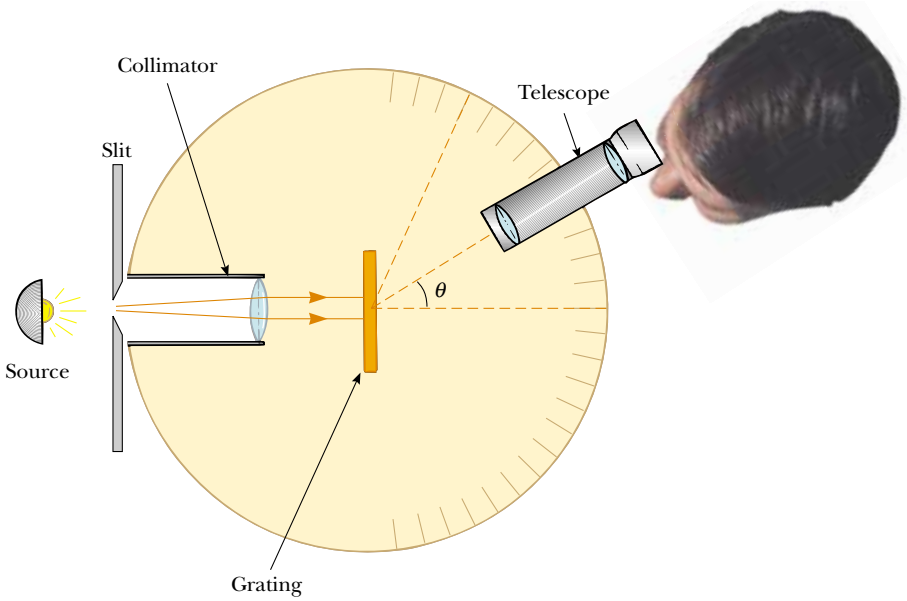


Figure 38.21 Diagram of a diffraction grating spectrometer. The collimated beam incident on the grating is diffracted into the various orders at the angles θ that satisfy the equation $d \sin \theta = m\lambda$, where $m = 0, 1, 2, \ldots$.

Conceptual Example 38.6 A Compact Disc Is a Diffraction Grating

Light reflected from the surface of a compact disc is multicolored, as shown in Figure 38.22. The colors and their intensities depend on the orientation of the disc relative to the eye and relative to the light source. Explain how this works.

Solution The surface of a compact disc has a spiral grooved track (with adjacent grooves having a separation on the order of 1 μm). Thus, the surface acts as a reflection grating. The light reflecting from the regions between these closely spaced grooves interferes constructively only in certain directions that depend on the wavelength and on the direction of the incident light. Any one section of the disc serves as a diffraction grating for white light, sending different colors in different directions. The different colors you see when viewing one section change as the light source, the disc, or you move to change the angles of incidence or diffraction.

Figure 38.22 A compact disc observed under white light. The colors observed in the reflected light and their intensities depend on the orientation of the disc relative to the eye and relative to the light source.

Example 38.7 The Orders of a Diffraction Grating

Monochromatic light from a helium-neon laser ($\lambda = 632.8$ nm) is incident normally on a diffraction grating containing 6 000 lines per centimeter. Find the angles at which the first-order, second-order, and third-order maxima are observed.

Solution First, we must calculate the slit separation, which is equal to the inverse of the number of lines per centimeter:

$$d = \frac{1}{6\,000}\ \text{cm} = 1.667 \times 10^{-4}\ \text{cm} = 1\,667\ \text{nm}$$

For the first-order maximum ($m = 1$), we obtain

$$\sin\theta_1 = \frac{\lambda}{d} = \frac{632.8\ \text{nm}}{1\,667\ \text{nm}} = 0.379\,6$$

$$\theta_1 = 22.31^\circ$$

For the second-order maximum ($m = 2$), we find

$$\sin\theta_2 = \frac{2\lambda}{d} = \frac{2(632.8\ \text{nm})}{1\,667\ \text{nm}} = 0.759\,2$$

$$\theta_2 = 49.39^\circ$$

For $m = 3$, we find that $\sin\theta_3 = 1.139$. Because $\sin\theta$ cannot exceed unity, this does not represent a realistic solution. Hence, only zeroth-, first-, and second-order maxima are observed for this situation.

Resolving Power of the Diffraction Grating

The diffraction grating is most useful for measuring wavelengths accurately. Like the prism, the diffraction grating can be used to disperse a spectrum into its wavelength components. Of the two devices, the grating is the more precise if one wants to distinguish two closely spaced wavelengths.

For two nearly equal wavelengths λ_1 and λ_2 between which a diffraction grating can just barely distinguish, the **resolving power** R of the grating is defined as

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda} \qquad (38.11)$$

Resolving power

where $\lambda = (\lambda_1 + \lambda_2)/2$ and $\Delta\lambda = \lambda_2 - \lambda_1$. Thus, a grating that has a high resolving power can distinguish small differences in wavelength. If N lines of the grating

are illuminated, it can be shown that the resolving power in the mth-order diffraction is

Resolving power of a grating

$$R = Nm \tag{38.12}$$

Thus, resolving power increases with increasing order number and with increasing number of illuminated slits.

Note that $R = 0$ for $m = 0$; this signifies that all wavelengths are indistinguishable for the zeroth-order maximum. However, consider the second-order diffraction pattern ($m = 2$) of a grating that has 5 000 rulings illuminated by the light source. The resolving power of such a grating in second order is $R = 5\,000 \times 2 = 10\,000$. Therefore, for a mean wavelength of, for example, 600 nm, the minimum wavelength separation between two spectral lines that can be just resolved is $\Delta\lambda = \lambda/R = 6.00 \times 10^{-2}$ nm. For the third-order principal maximum, $R = 15\,000$ and $\Delta\lambda = 4.00 \times 10^{-2}$ nm, and so on.

One of the most interesting applications of diffraction is holography, which is used to create three-dimensional images found practically everywhere, from credit cards to postage stamps. The production of these special diffracting films is discussed in Chapter 42 of the extended version of this text.

EXAMPLE 38.8 Resolving Sodium Spectral Lines

When an element is raised to a very high temperature, the atoms emit radiation having discrete wavelengths. The set of wavelengths for a given element is called its *atomic spectrum*. Two strong components in the atomic spectrum of sodium have wavelengths of 589.00 nm and 589.59 nm. (a) What must be the resolving power of a grating if these wavelengths are to be distinguished?

Solution

$$R = \frac{\lambda}{\Delta\lambda} = \frac{589.30 \text{ nm}}{589.59 \text{ nm} - 589.00 \text{ nm}} = \frac{589.30}{0.59} = 999$$

(b) To resolve these lines in the second-order spectrum, how many lines of the grating must be illuminated?

Solution From Equation 38.12 and the results to part (a), we find that

$$N = \frac{R}{m} = \frac{999}{2} = 500 \text{ lines}$$

Optional Section

38.5 DIFFRACTION OF X-RAYS BY CRYSTALS

In principle, the wavelength of any electromagnetic wave can be determined if a grating of the proper spacing (of the order of λ) is available. X-rays, discovered by Wilhelm Roentgen (1845–1923) in 1895, are electromagnetic waves of very short wavelength (of the order of 0.1 nm). It would be impossible to construct a grating having such a small spacing by the cutting process described at the beginning of Section 38.4. However, the atomic spacing in a solid is known to be about 0.1 nm. In 1913, Max von Laue (1879–1960) suggested that the regular array of atoms in a crystal could act as a three-dimensional diffraction grating for x-rays. Subsequent experiments confirmed this prediction. The diffraction patterns are complex because of the three-dimensional nature of the crystal. Nevertheless, x-ray diffraction

has proved to be an invaluable technique for elucidating crystalline structures and for understanding the structure of matter.[1]

Figure 38.23 is one experimental arrangement for observing x-ray diffraction from a crystal. A collimated beam of x-rays is incident on a crystal. The diffracted beams are very intense in certain directions, corresponding to constructive interference from waves reflected from layers of atoms in the crystal. The diffracted beams can be detected by a photographic film, and they form an array of spots known as a *Laue pattern*. One can deduce the crystalline structure by analyzing the positions and intensities of the various spots in the pattern.

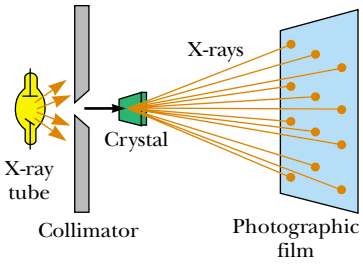


Figure 38.23 Schematic diagram of the technique used to observe the diffraction of x-rays by a crystal. The array of spots formed on the film is called a Laue pattern.

The arrangement of atoms in a crystal of sodium chloride (NaCl) is shown in Figure 38.24. Each unit cell (the geometric solid that repeats throughout the crystal) is a cube having an edge length a. A careful examination of the NaCl structure shows that the ions lie in discrete planes (the shaded areas in Fig. 38.24). Now suppose that an incident x-ray beam makes an angle θ with one of the planes, as shown in Figure 38.25. The beam can be reflected from both the upper plane and the lower one. However, the beam reflected from the lower plane travels farther than the beam reflected from the upper plane. The effective path difference is $2d \sin \theta$. The two beams reinforce each other (constructive interference) when this path difference equals some integer multiple of λ. The same is true for reflection from the entire family of parallel planes. Hence, the condition for constructive interference (maxima in the reflected beam) is

$$2d \sin \theta = m\lambda \qquad m = 1, 2, 3, \ldots \tag{38.13}$$

Bragg's law

This condition is known as **Bragg's law,** after W. L. Bragg (1890–1971), who first derived the relationship. If the wavelength and diffraction angle are measured, Equation 38.13 can be used to calculate the spacing between atomic planes.

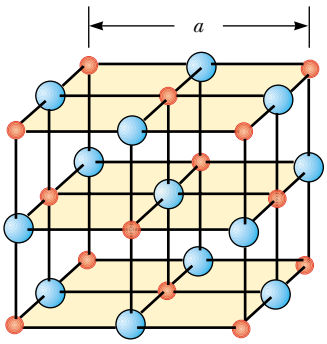


Figure 38.24 Crystalline structure of sodium chloride (NaCl). The blue spheres represent Cl^- ions, and the red spheres represent Na^+ ions. The length of the cube edge is $a = 0.562\ 737$ nm.

Quick Quiz 38.3

When you receive a chest x-ray at a hospital, the rays pass through a series of parallel ribs in your chest. Do the ribs act as a diffraction grating for x-rays?

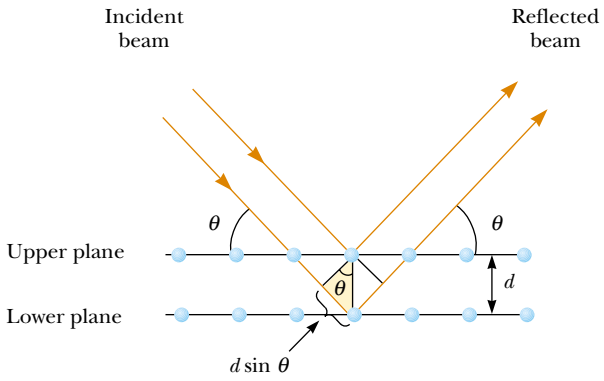


Figure 38.25 A two-dimensional description of the reflection of an x-ray beam from two parallel crystalline planes separated by a distance d. The beam reflected from the lower plane travels farther than the one reflected from the upper plane by a distance $2d \sin \theta$.

[1] For more details on this subject, see Sir Lawrence Bragg, "X-Ray Crystallography," *Sci. Am.* 219:58–70, 1968.

38.6 POLARIZATION OF LIGHT WAVES

In Chapter 34 we described the transverse nature of light and all other electromagnetic waves. Polarization is firm evidence of this transverse nature.

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a wave having some particular orientation of the electric field vector **E**, corresponding to the direction of atomic vibration. The *direction of polarization* of each individual wave is defined to be the direction in which the electric field is vibrating. In Figure 38.26, this direction happens to lie along the *y* axis. However, an individual electromagnetic wave could have its **E** vector in the *yz* plane, making any possible angle with the *y* axis. Because all directions of vibration from a wave source are possible, the resultant electromagnetic wave is a superposition of waves vibrating in many different directions. The result is an **unpolarized** light beam, represented in Figure 38.27a. The direction of wave propagation in this figure is perpendicular to the page. The arrows show a few possible directions of the electric field vectors for the individual waves making up the resultant beam. At any given point and at some instant of time, all these individual electric field vectors add to give one resultant electric field vector.

As noted in Section 34.2, a wave is said to be **linearly polarized** if the resultant electric field **E** vibrates in the same direction *at all times* at a particular point, as shown in Figure 38.27b. (Sometimes, such a wave is described as *plane-polarized,* or simply *polarized.*) The plane formed by **E** and the direction of propagation is called the *plane of polarization* of the wave. If the wave in Figure 38.26 represented the resultant of all individual waves, the plane of polarization is the *xy* plane.

It is possible to obtain a linearly polarized beam from an unpolarized beam by removing all waves from the beam except those whose electric field vectors oscillate in a single plane. We now discuss four processes for producing polarized light from unpolarized light.

Polarization by Selective Absorption

The most common technique for producing polarized light is to use a material that transmits waves whose electric fields vibrate in a plane parallel to a certain direction and that absorbs waves whose electric fields vibrate in all other directions.

In 1938, E. H. Land (1909–1991) discovered a material, which he called *polaroid,* that polarizes light through selective absorption by oriented molecules. This material is fabricated in thin sheets of long-chain hydrocarbons. The sheets are stretched during manufacture so that the long-chain molecules align. After a sheet is dipped into a solution containing iodine, the molecules become good electrical conductors. However, conduction takes place primarily along the hydrocarbon chains because electrons can move easily only along the chains. As a result, the

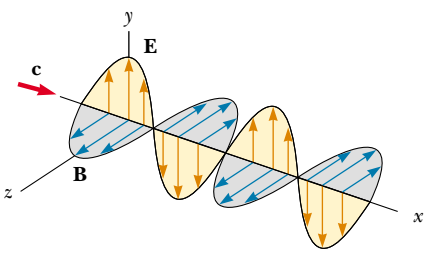


Figure 38.26 Schematic diagram of an electromagnetic wave propagating at velocity **c** in the *x* direction. The electric field vibrates in the *xy* plane, and the magnetic field vibrates in the *xz* plane.

molecules readily absorb light whose electric field vector is parallel to their length and allow light through whose electric field vector is perpendicular to their length.

It is common to refer to the direction perpendicular to the molecular chains as the *transmission axis*. In an ideal polarizer, all light with **E** parallel to the transmission axis is transmitted, and all light with **E** perpendicular to the transmission axis is absorbed.

Figure 38.28 represents an unpolarized light beam incident on a first polarizing sheet, called the *polarizer*. Because the transmission axis is oriented vertically in the figure, the light transmitted through this sheet is polarized vertically. A second polarizing sheet, called the *analyzer*, intercepts the beam. In Figure 38.28, the analyzer transmission axis is set at an angle θ to the polarizer axis. We call the electric field vector of the transmitted beam $\mathbf{E}_0$. The component of $\mathbf{E}_0$ perpendicular to the analyzer axis is completely absorbed. The component of $\mathbf{E}_0$ parallel to the analyzer axis, which is allowed through by the analyzer, is $E_0 \cos\theta$. Because the intensity of the transmitted beam varies as the square of its magnitude, we conclude that the intensity of the (polarized) beam transmitted through the analyzer varies as

$$I = I_{\max} \cos^2 \theta \qquad \textbf{(38.14)}$$

where $I_{\max}$ is the intensity of the polarized beam incident on the analyzer. This expression, known as **Malus's law,**[2] applies to any two polarizing materials whose transmission axes are at an angle θ to each other. From this expression, note that the intensity of the transmitted beam is maximum when the transmission axes are parallel ($\theta = 0$ or $180°$) and that it is zero (complete absorption by the analyzer) when the transmission axes are perpendicular to each other. This variation in transmitted intensity through a pair of polarizing sheets is illustrated in Figure 38.29. Because the average value of $\cos^2\theta$ is $\frac{1}{2}$, the intensity of the light passed through an ideal polarizer is one-half the intensity of unpolarized light.

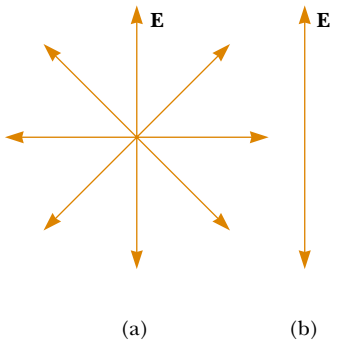


Figure 38.27 (a) An unpolarized light beam viewed along the direction of propagation (perpendicular to the page). The transverse electric field can vibrate in any direction in the plane of the page with equal probability. (b) A linearly polarized light beam with the electric field vibrating in the vertical direction.

Polarization by Reflection

When an unpolarized light beam is reflected from a surface, the reflected light may be completely polarized, partially polarized, or unpolarized, depending on the angle of incidence. If the angle of incidence is 0°, the reflected beam is unpolarized. For other angles of incidence, the reflected light is polarized to some ex-

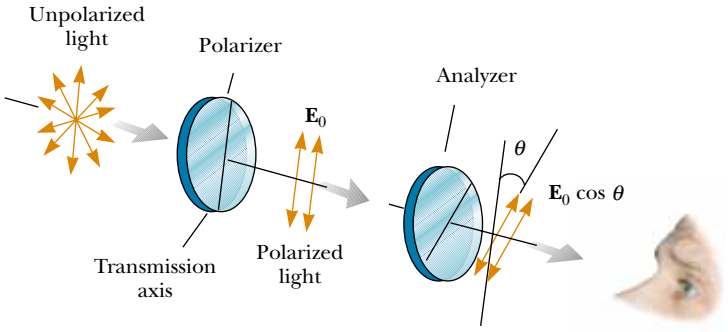
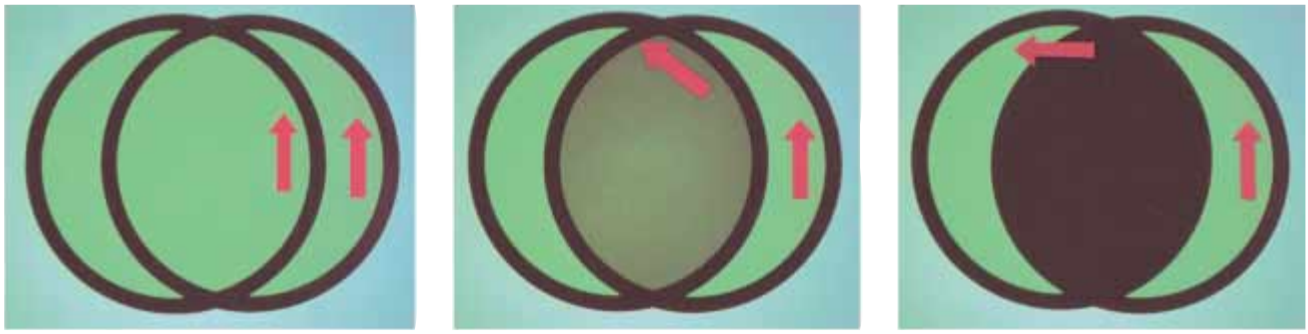


Figure 38.28 Two polarizing sheets whose transmission axes make an angle θ with each other. Only a fraction of the polarized light incident on the analyzer is transmitted through it.

[2] Named after its discoverer, E. L. Malus (1775–1812). Malus discovered that reflected light was polarized by viewing it through a calcite ($CaCO_3$) crystal.



(a) (b) (c)

Figure 38.29 The intensity of light transmitted through two polarizers depends on the relative orientation of their transmission axes. (a) The transmitted light has maximum intensity when the transmission axes are aligned with each other. (b) The transmitted light has lesser intensity when the transmission axes are at an angle of 45° with each other. (c) The transmitted light intensity is a minimum when the transmission axes are at right angles to each other.

tent, and for one particular angle of incidence, the reflected light is completely polarized. Let us now investigate reflection at that special angle.

Suppose that an unpolarized light beam is incident on a surface, as shown in Figure 38.30a. Each individual electric field vector can be resolved into two components: one parallel to the surface (and perpendicular to the page in Fig. 38.30, represented by the dots), and the other (represented by the red arrows) perpendicular both to the first component and to the direction of propagation. Thus, the polarization of the entire beam can be described by two electric field components in these directions. It is found that the parallel component reflects more strongly than the perpendicular component, and this results in a partially polarized reflected beam. Furthermore, the refracted beam is also partially polarized.

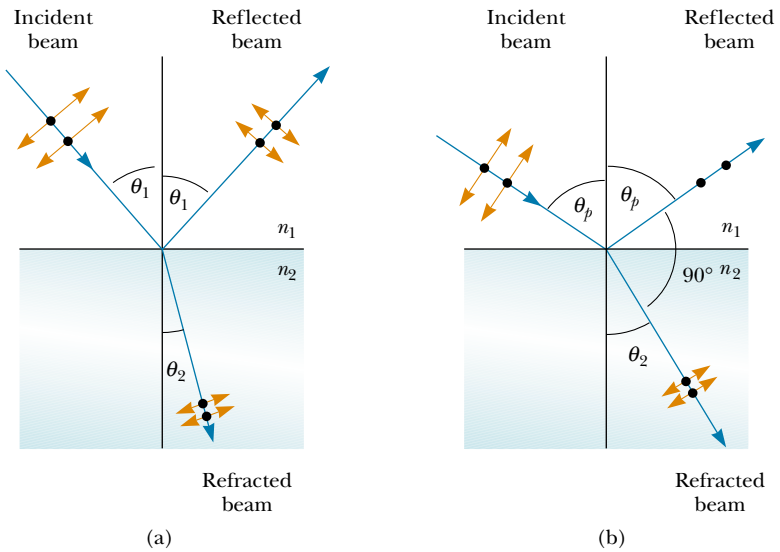


Figure 38.30 (a) When unpolarized light is incident on a reflecting surface, the reflected and refracted beams are partially polarized. (b) The reflected beam is completely polarized when the angle of incidence equals the polarizing angle θ_p, which satisfies the equation $n = \tan \theta_p$.

Now suppose that the angle of incidence θ_1 is varied until the angle between the reflected and refracted beams is 90°, as shown in Figure 38.30b. At this particular angle of incidence, the reflected beam is completely polarized (with its electric field vector parallel to the surface), and the refracted beam is still only partially polarized. The angle of incidence at which this polarization occurs is called the **polarizing angle** θ_p.

Polarizing angle

We can obtain an expression relating the polarizing angle to the index of refraction of the reflecting substance by using Figure 38.30b. From this figure, we see that $\theta_p + 90° + \theta_2 = 180°$; thus, $\theta_2 = 90° - \theta_p$. Using Snell's law of refraction (Eq. 35.8) and taking $n_1 = 1.00$ for air and $n_2 = n$, we have

$$n = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\sin \theta_p}{\sin \theta_2}$$

Because $\sin \theta_2 = \sin(90° - \theta_p) = \cos \theta_p$, we can write this expression for n as $n = \sin \theta_p / \cos \theta_p$, which means that

$$n = \tan \theta_p \qquad \textbf{(38.15)}$$

Brewster's law

This expression is called **Brewster's law,** and the polarizing angle θ_p is sometimes called **Brewster's angle,** after its discoverer, David Brewster (1781–1868). Because n varies with wavelength for a given substance, Brewster's angle is also a function of wavelength.

Polarization by reflection is a common phenomenon. Sunlight reflected from water, glass, and snow is partially polarized. If the surface is horizontal, the electric field vector of the reflected light has a strong horizontal component. Sunglasses made of polarizing material reduce the glare of reflected light. The transmission axes of the lenses are oriented vertically so that they absorb the strong horizontal component of the reflected light. If you rotate sunglasses 90°, they will not be as effective at blocking the glare from shiny horizontal surfaces.

QuickLab

Devise a way to use a protractor, desklamp, and polarizing sunglasses to measure Brewster's angle for the glass in a window. From this, determine the index of refraction of the glass. Compare your results with the values given in Table 35.1.

Polarization by Double Refraction

Solids can be classified on the basis of internal structure. Those in which the atoms are arranged in a specific order are called *crystalline;* the NaCl structure of Figure 38.24 is just one example of a crystalline solid. Those solids in which the atoms are distributed randomly are called *amorphous.* When light travels through an amorphous material, such as glass, it travels with a speed that is the same in all directions. That is, glass has a single index of refraction. In certain crystalline materials, however, such as calcite and quartz, the speed of light is not the same in all directions. Such materials are characterized by two indices of refraction. Hence, they are often referred to as **double-refracting** or **birefringent** materials.

Upon entering a calcite crystal, unpolarized light splits into two plane-polarized rays that travel with different velocities, corresponding to two angles of refraction, as shown in Figure 38.31. The two rays are polarized in two mutually perpendicular directions, as indicated by the dots and arrows. One ray, called the **ordinary (O) ray,** is characterized by an index of refraction n_O that is the same in all directions. This means that if one could place a point source of light inside the crystal, as shown in Figure 38.32, the ordinary waves would spread out from the source as spheres.

The second plane-polarized ray, called the **extraordinary (E) ray,** travels with different speeds in different directions and hence is characterized by an index of refraction n_E that varies with the direction of propagation. The point source in Fig-

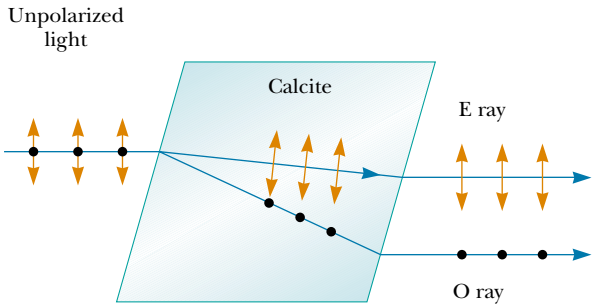


Figure 38.31 Unpolarized light incident on a calcite crystal splits into an ordinary (O) ray and an extraordinary (E) ray. These two rays are polarized in mutually perpendicular directions (drawing not to scale).

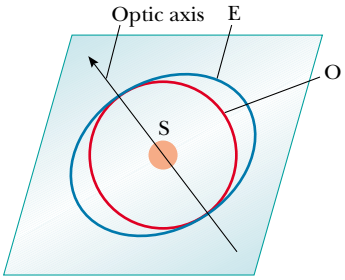


Figure 38.32 A point source S inside a double-refracting crystal produces a spherical wave front corresponding to the ordinary ray and an elliptical wave front corresponding to the extraordinary ray. The two waves propagate with the same velocity along the optic axis.

ure 38.32 sends out an extraordinary wave having wave fronts that are elliptical in cross-section. Note from Figure 38.32 that there is one direction, called the **optic axis,** along which the ordinary and extraordinary rays have the same speed, corresponding to the direction for which $n_O = n_E$. The difference in speed for the two rays is a maximum in the direction perpendicular to the optic axis. For example, in calcite, $n_O = 1.658$ at a wavelength of 589.3 nm, and n_E varies from 1.658 along the optic axis to 1.486 perpendicular to the optic axis. Values for n_O and n_E for various double-refracting crystals are given in Table 38.1.

If we place a piece of calcite on a sheet of paper and then look through the crystal at any writing on the paper, we see two images, as shown in Figure 38.33. As can be seen from Figure 38.31, these two images correspond to one formed by the ordinary ray and one formed by the extraordinary ray. If the two images are viewed through a sheet of rotating polarizing glass, they alternately appear and disappear because the ordinary and extraordinary rays are plane-polarized along mutually perpendicular directions.

Polarization by Scattering

When light is incident on any material, the electrons in the material can absorb and reradiate part of the light. Such absorption and reradiation of light by electrons in the gas molecules that make up air is what causes sunlight reaching an observer on the Earth to be partially polarized. You can observe this effect—called **scattering**—by looking directly up at the sky through a pair of sunglasses whose lenses are made of polarizing material. Less light passes through at certain orientations of the lenses than at others.

Figure 38.34 illustrates how sunlight becomes polarized when it is scattered. An unpolarized beam of sunlight traveling in the horizontal direction (parallel to

Figure 38.33 A calcite crystal produces a double image because it is a birefringent (double-refracting) material.

TABLE 38.1 **Indices of Refraction for Some Double-Refracting Crystals at a Wavelength of 589.3 nm**

Crystal	n_O	n_E	n_O/n_E
Calcite ($CaCO_3$)	1.658	1.486	1.116
Quartz (SiO_2)	1.544	1.553	0.994
Sodium nitrate ($NaNO_3$)	1.587	1.336	1.188
Sodium sulfite ($NaSO_3$)	1.565	1.515	1.033
Zinc chloride ($ZnCl_2$)	1.687	1.713	0.985
Zinc sulfide (ZnS)	2.356	2.378	0.991

the ground) strikes a molecule of one of the gases that make up air, setting the electrons of the molecule into vibration. These vibrating charges act like the vibrating charges in an antenna. The horizontal component of the electric field vector in the incident wave results in a horizontal component of the vibration of the charges, and the vertical component of the vector results in a vertical component of vibration. If the observer in Figure 38.34 is looking straight up (perpendicular to the original direction of propagation of the light), the vertical oscillations of the charges send no radiation toward the observer. Thus, the observer sees light that is completely polarized in the horizontal direction, as indicated by the red arrows. If the observer looks in other directions, the light is partially polarized in the horizontal direction.

Some phenomena involving the scattering of light in the atmosphere can be understood as follows. When light of various wavelengths λ is incident on gas molecules of diameter d, where $d \ll \lambda$, the relative intensity of the scattered light varies as $1/\lambda^4$. The condition $d \ll \lambda$ is satisfied for scattering from oxygen (O_2) and nitrogen (N_2) molecules in the atmosphere, whose diameters are about 0.2 nm. Hence, short wavelengths (blue light) are scattered more efficiently than long wavelengths (red light). Therefore, when sunlight is scattered by gas molecules in the air, the short-wavelength radiation (blue) is scattered more intensely than the long-wavelength radiation (red).

When you look up into the sky in a direction that is not toward the Sun, you see the scattered light, which is predominantly blue; hence, you see a blue sky. If you look toward the west at sunset (or toward the east at sunrise), you are looking in a direction toward the Sun and are seeing light that has passed through a large distance of air. Most of the blue light has been scattered by the air between you and the Sun. The light that survives this trip through the air to you has had much of its blue component scattered and is thus heavily weighted toward the red end of the spectrum; as a result, you see the red and orange colors of sunset. However, a blue sky is seen by someone to your west for whom it is still a quarter hour before sunset.

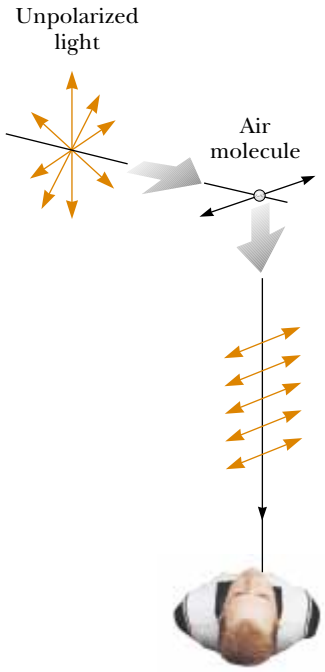


Figure 38.34 The scattering of unpolarized sunlight by air molecules. The scattered light traveling perpendicular to the incident light is plane-polarized because the vertical vibrations of the charges in the air molecule send no light in this direction.

Optical Activity

Many important applications of polarized light involve materials that display **optical activity.** A material is said to be optically active if it rotates the plane of polarization of any light transmitted through the material. The angle through which the light is rotated by a specific material depends on the length of the path through the material and on concentration if the material is in solution. One optically active material is a solution of the common sugar dextrose. A standard method for determining the concentration of sugar solutions is to measure the rotation produced by a fixed length of the solution.

Molecular asymmetry determines whether a material is optically active. For example, some proteins are optically active because of their spiral shape. Other materials, such as glass and plastic, become optically active when stressed. Suppose that an unstressed piece of plastic is placed between a polarizer and an analyzer so that light passes from polarizer to plastic to analyzer. When the plastic is unstressed and the analyzer axis is perpendicular to the polarizer axis, none of the polarized light passes through the analyzer. In other words, the unstressed plastic has no effect on the light passing through it. If the plastic is stressed, however, the regions of greatest stress rotate the polarized light through the largest angles. Hence, a series of bright and dark bands is observed in the transmitted light, with the bright bands corresponding to regions of greatest stress.

(a) (b)

Figure 38.35 (a) Strain distribution in a plastic model of a hip replacement used in a medical research laboratory. The pattern is produced when the plastic model is viewed between a polarizer and analyzer oriented perpendicular to each other. (b) A plastic model of an arch structure under load conditions observed between perpendicular polarizers. Such patterns are useful in the optimum design of architectural components.

Engineers often use this technique, called *optical stress analysis,* in designing structures ranging from bridges to small tools. They build a plastic model and analyze it under different load conditions to determine regions of potential weakness and failure under stress. Some examples of a plastic model under stress are shown in Figure 38.35.

The liquid crystal displays found in most calculators have their optical activity changed by the application of electric potential across different parts of the display. Try using a pair of polarizing sunglasses to investigate the polarization used in the display of your calculator.

Summary

Diffraction is the deviation of light from a straight-line path when the light passes through an aperture or around an obstacle.

The **Fraunhofer diffraction pattern** produced by a single slit of width a on a distant screen consists of a central bright fringe and alternating bright and dark fringes of much lower intensities. The angles θ at which the diffraction pattern has zero intensity, corresponding to destructive interference, are given by

$$\sin\theta = m\frac{\lambda}{a} \qquad m = \pm 1, \pm 2, \pm 3, \ldots \tag{38.1}$$

How the intensity I of a single-slit diffraction pattern varies with angle θ is given by the expression

$$I = I_{\max}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 \tag{38.4}$$

where $\beta = (2\pi a \sin\theta)/\lambda$ and $I_{\max}$ is the intensity at $\theta = 0$.

Rayleigh's criterion, which is a limiting condition of resolution, states that two images formed by an aperture are just distinguishable if the central maximum of the diffraction pattern for one image falls on the first minimum of the diffrac-

tion pattern for the other image. The limiting angle of resolution for a slit of width a is $\theta_{\min} = \lambda/a$, and the limiting angle of resolution for a circular aperture of diameter D is $\theta_{\min} = 1.22\lambda/D$.

A **diffraction grating** consists of a large number of equally spaced, identical slits. The condition for intensity maxima in the interference pattern of a diffraction grating for normal incidence is

$$d \sin \theta = m\lambda \qquad m = 0, 1, 2, 3, \ldots \tag{38.10}$$

where d is the spacing between adjacent slits and m is the order number of the diffraction pattern. The resolving power of a diffraction grating in the mth order of the diffraction pattern is

$$R = Nm \tag{38.12}$$

where N is the number of lines in the grating that are illuminated.

When polarized light of intensity I_0 is emitted by a polarizer and then incident on an analyzer, the light transmitted through the analyzer has an intensity equal to $I_{\max} \cos^2 \theta$, where θ is the angle between the polarizer and analyzer transmission axes.

In general, reflected light is partially polarized. However, reflected light is completely polarized when the angle of incidence is such that the angle between the reflected and refracted beams is 90°. This angle of incidence, called the **polarizing angle** θ_p, satisfies **Brewster's law:**

$$n = \tan \theta_p \tag{38.15}$$

where n is the index of refraction of the reflecting medium.

Questions

1. Why can you hear around corners but not see around them?
2. Observe the shadow of your book when it is held a few inches above a table while illuminated by a lamp several feet above it. Why is the shadow somewhat fuzzy at the edges?
3. Knowing that radio waves travel at the speed of light and that a typical AM radio frequency is 1 000 kHz while an FM radio frequency might be 100 MHz, estimate the wavelengths of typical AM and FM radio signals. Use this information to explain why FM radio stations often fade out when you drive through a short tunnel or underpass but AM radio stations do not.
4. Describe the change in width of the central maximum of the single-slit diffraction pattern as the width of the slit is made narrower.
5. Assuming that the headlights of a car are point sources, estimate the maximum observer-to-car distance at which the headlights are distinguishable from each other.
6. A laser beam is incident at a shallow angle on a machinist's ruler that has a finely calibrated scale. The engraved rulings on the scale give rise to a diffraction pattern on a screen. Discuss how you can use this arrangement to obtain a measure of the wavelength of the laser light.
7. Certain sunglasses use a polarizing material to reduce the intensity of light reflected from shiny surfaces. What orientation of polarization should the material have to be most effective?
8. During the "day" on the Moon (that is, when the Sun is visible), you see a black sky and the stars are clearly visible. During the day on the Earth, you see a blue sky and no stars. Account for this difference.
9. You can make the path of a light beam visible by placing dust in the air (perhaps by shaking a blackboard eraser in the path of the light beam). Explain why you can see the beam under these circumstances.
10. Is light from the sky polarized? Why is it that clouds seen through Polaroid glasses stand out in bold contrast to the sky?
11. If a coin is glued to a glass sheet and the arrangement is held in front of a laser beam, the projected shadow has diffraction rings around its edge and a bright spot in the center. How is this possible?
12. If a fine wire is stretched across the path of a laser beam, is it possible to produce a diffraction pattern?
13. How could the index of refraction of a flat piece of dark obsidian glass be determined?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
= paired numerical/symbolic problems

Section 38.1 Introduction to Diffraction

Section 38.2 Diffraction from Narrow Slits

1. Helium-neon laser light ($\lambda = 632.8$ nm) is sent through a 0.300-mm-wide single slit. What is the width of the central maximum on a screen 1.00 m from the slit?
2. A beam of green light is diffracted by a slit with a width of 0.550 mm. The diffraction pattern forms on a wall 2.06 m beyond the slit. The distance between the positions of zero intensity on both sides of the central bright fringe is 4.10 mm. Calculate the wavelength of the laser light.
3. WEB A screen is placed 50.0 cm from a single slit, which is illuminated with 690-nm light. If the distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit?
4. Coherent microwaves of wavelength 5.00 cm enter a long, narrow window in a building otherwise essentially opaque to the microwaves. If the window is 36.0 cm wide, what is the distance from the central maximum to the first-order minimum along a wall 6.50 m from the window?
5. Sound with a frequency of 650 Hz from a distant source passes through a doorway 1.10 m wide in a sound-absorbing wall. Find the number and approximate directions of the diffraction-maximum beams radiated into the space beyond.
6. Light with a wavelength of 587.5 nm illuminates a single slit 0.750 mm in width. (a) At what distance from the slit should a screen be located if the first minimum in the diffraction pattern is to be 0.850 mm from the center of the screen? (b) What is the width of the central maximum?
7. A diffraction pattern is formed on a screen 120 cm away from a 0.400-mm-wide slit. Monochromatic 546.1-nm light is used. Calculate the fractional intensity I/I_0 at a point on the screen 4.10 mm from the center of the principal maximum.
8. The second-order bright fringe in a single-slit diffraction pattern is 1.40 mm from the center of the central maximum. The screen is 80.0 cm from a slit of width 0.800 mm. Assuming that the incident light is monochromatic, calculate the light's approximate wavelength.
9. If the light in Figure 38.5 strikes the single slit at an angle β from the perpendicular direction, show that Equation 38.1, the condition for destructive interference, must be modified to read

$$\sin\theta = m\left(\frac{\lambda}{a}\right) - \sin\beta$$

10. Coherent light with a wavelength of 501.5 nm is sent through two parallel slits in a large flat wall. Each slit is 0.700 μm wide, and the slits' centers are 2.80 μm apart. The light falls on a semicylindrical screen, with its axis at the midline between the slits. (a) Predict the direction of each interference maximum on the screen, as an angle away from the bisector of the line joining the slits. (b) Describe the pattern of light on the screen, specifying the number of bright fringes and the location of each. (c) Find the intensity of light on the screen at the center of each bright fringe, expressed as a fraction of the light intensity I_0 at the center of the pattern.

Section 38.3 Resolution of Single-Slit and Circular Apertures

11. The pupil of a cat's eye narrows to a vertical slit of width 0.500 mm in daylight. What is the angular resolution for horizontally separated mice? Assume that the average wavelength of the light is 500 nm.
12. Find the radius of a star image formed on the retina of the eye if the aperture diameter (the pupil) at night is 0.700 cm and the length of the eye is 3.00 cm. Assume that the representative wavelength of starlight in the eye is 500 nm.
13. WEB A helium-neon laser emits light that has a wavelength of 632.8 nm. The circular aperture through which the beam emerges has a diameter of 0.500 cm. Estimate the diameter of the beam 10.0 km from the laser.
14. On the night of April 18, 1775, a signal was to be sent from the steeple of Old North Church in Boston to Paul Revere, who was 1.80 mi away: "One if by land, two if by sea." At what minimum separation did the sexton have to set the lanterns for Revere to receive the correct message? Assume that Revere's pupils had a diameter of 4.00 mm at night and that the lantern light had a predominant wavelength of 580 nm.
15. The Impressionist painter Georges Seurat created paintings with an enormous number of dots of pure pigment, each of which was approximately 2.00 mm in diameter. The idea was to locate colors such as red and green next to each other to form a scintillating canvas (Fig. P38.15). Outside what distance would one be unable to discern individual dots on the canvas? (Assume that $\lambda = 500$ nm and that the pupil diameter is 4.00 mm.)
16. A binary star system in the constellation Orion has an angular interstellar separation of 1.00×10^{-5} rad. If $\lambda = 500$ nm, what is the smallest diameter a telescope must have to just resolve the two stars?

Figure P38.15 *Sunday Afternoon on the Isle of La Grande Jatte,* by Georges Seurat. *(SuperStock)*

17. A child is standing at the edge of a straight highway watching her grandparents' car driving away at 20.0 m/s. The air is perfectly clear and steady, and after 10.0 min the car's two taillights appear to merge into one. Assuming the diameter of the child's pupils is 5.00 mm, estimate the width of the car.

18. Suppose that you are standing on a straight highway and watching a car moving away from you at a speed v. The air is perfectly clear and steady, and after a time t the taillights appear to merge into one. Assuming the diameter of your pupil is d, estimate the width of the car.

19. A circular radar antenna on a Coast Guard ship has a diameter of 2.10 m and radiates at a frequency of 15.0 GHz. Two small boats are located 9.00 km away from the ship. How close together could the boats be and still be detected as two objects?

20. If we were to send a ruby laser beam ($\lambda = 694.3$ nm) outward from the barrel of a 2.70-m-diameter telescope, what would be the diameter of the big red spot when the beam hit the Moon 384 000 km away? (Neglect atmospheric dispersion.)

21. The angular resolution of a radio telescope is to be 0.100° when the incident waves have a wavelength of 3.00 mm. What minimum diameter is required for the telescope's receiving dish?

22. When Mars is nearest the Earth, the distance separating the two planets is 88.6×10^6 km. Mars is viewed through a telescope whose mirror has a diameter of 30.0 cm. (a) If the wavelength of the light is 590 nm, what is the angular resolution of the telescope? (b) What is the smallest distance that can be resolved between two points on Mars?

Section 38.4 The Diffraction Grating

Note: In the following problems, assume that the light is incident normally on the gratings.

23. White light is spread out into its spectral components by a diffraction grating. If the grating has 2 000 lines per centimeter, at what angle does red light of wavelength 640 nm appear in first order?

24. Light from an argon laser strikes a diffraction grating that has 5 310 lines per centimeter. The central and first-order principal maxima are separated by 0.488 m on a wall 1.72 m from the grating. Determine the wavelength of the laser light.

WEB **25.** The hydrogen spectrum has a red line at 656 nm and a violet line at 434 nm. What is the angular separation between two spectral lines obtained with a diffraction grating that has 4 500 lines per centimeter?

26. A helium-neon laser ($\lambda = 632.8$ nm) is used to calibrate a diffraction grating. If the first-order maximum occurs at 20.5°, what is the spacing between adjacent grooves in the grating?

27. Three discrete spectral lines occur at angles of 10.09°, 13.71°, and 14.77° in the first-order spectrum of a grating spectroscope. (a) If the grating has 3 660 slits per centimeter, what are the wavelengths of the light? (b) At what angles are these lines found in the second-order spectrum?

28. A diffraction grating has 800 rulings per millimeter. A beam of light containing wavelengths from 500 to 700 nm hits the grating. Do the spectra of different orders overlap? Explain.

WEB **29.** A diffraction grating with a width of 4.00 cm has been ruled with 3 000 grooves per centimeter. (a) What is the resolving power of this grating in the first three orders? (b) If two monochromatic waves incident on this grating have a mean wavelength of 400 nm, what is their wavelength separation if they are just resolved in the third order?

30. Show that, whenever white light is passed through a diffraction grating of any spacing size, the violet end of the continuous visible spectrum in third order always overlaps the red light at the other end of the second-order spectrum.

31. A source emits 531.62-nm and 531.81-nm light. (a) What minimum number of lines is required for a grating that resolves the two wavelengths in the first-order spectrum? (b) Determine the slit spacing for a grating 1.32 cm wide that has the required minimum number of lines.

32. Two wavelengths λ and $\lambda + \Delta\lambda$ (with $\Delta\lambda \ll \lambda$) are incident on a diffraction grating. Show that the angular separation between the spectral lines in the mth order spectrum is

$$\Delta\theta = \frac{\Delta\lambda}{\sqrt{(d/m)^2 - \lambda^2}}$$

where d is the slit spacing and m is the order number.

33. A grating with 250 lines per millimeter is used with an incandescent light source. Assume that the visible spectrum ranges in wavelength from 400 to 700 nm. In how

many orders can one see (a) the entire visible spectrum and (b) the short-wavelength region?

34. A diffraction grating has 4 200 rulings per centimeter. On a screen 2.00 m from the grating, it is found that for a particular order m, the maxima corresponding to two closely spaced wavelengths of sodium (589.0 nm and 589.6 nm) are separated by 1.59 mm. Determine the value of m.

(Optional)

Section 38.5 Diffraction of X-Rays by Crystals

35. Potassium iodide (KI) has the same crystalline structure as NaCl, with $d = 0.353$ nm. A monochromatic x-ray beam shows a diffraction maximum when the grazing angle is 7.60°. Calculate the x-ray wavelength. (Assume first order.)

36. A wavelength of 0.129 nm characterizes K_α x-rays from zinc. When a beam of these x-rays is incident on the surface of a crystal whose structure is similar to that of NaCl, a first-order maximum is observed at 8.15°. Calculate the interplanar spacing on the basis of this information.

WEB **37.** If the interplanar spacing of NaCl is 0.281 nm, what is the predicted angle at which 0.140-nm x-rays are diffracted in a first-order maximum?

38. The first-order diffraction maximum is observed at 12.6° for a crystal in which the interplanar spacing is 0.240 nm. How many other orders can be observed?

39. Monochromatic x-rays of the K_α line from a nickel target ($\lambda = 0.166$ nm) are incident on a potassium chloride (KCl) crystal surface. The interplanar distance in KCl is 0.314 nm. At what angle (relative to the surface) should the beam be directed for a second-order maximum to be observed?

40. In water of uniform depth, a wide pier is supported on pilings in several parallel rows 2.80 m apart. Ocean waves of uniform wavelength roll in, moving in a direction that makes an angle of 80.0° with the rows of posts. Find the three longest wavelengths of waves that will be strongly reflected by the pilings.

Section 38.6 Polarization of Light Waves

41. Unpolarized light passes through two polaroid sheets. The axis of the first is vertical, and that of the second is at 30.0° to the vertical. What fraction of the initial light is transmitted?

42. Three polarizing disks whose planes are parallel are centered on a common axis. The direction of the transmission axis in each case is shown in Figure P38.42 relative to the common vertical direction. A plane-polarized beam of light with E_0 parallel to the vertical reference direction is incident from the left on the first disk with an intensity of $I_i = 10.0$ units (arbitrary). Calculate the transmitted intensity I_f when (a) $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, and $\theta_3 = 60.0°$; (b) $\theta_1 = 0°$, $\theta_2 = 30.0°$, and $\theta_3 = 60.0°$.

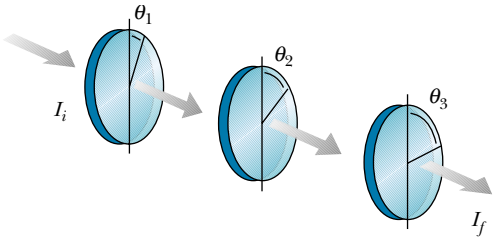


Figure P38.42 Problems 42 and 48.

43. Plane-polarized light is incident on a single polarizing disk with the direction of E_0 parallel to the direction of the transmission axis. Through what angle should the disk be rotated so that the intensity in the transmitted beam is reduced by a factor of (a) 3.00, (b) 5.00, (c) 10.0?

44. The angle of incidence of a light beam onto a reflecting surface is continuously variable. The reflected ray is found to be completely polarized when the angle of incidence is 48.0°. What is the index of refraction of the reflecting material?

45. The critical angle for total internal reflection for sapphire surrounded by air is 34.4°. Calculate the polarizing angle for sapphire.

46. For a particular transparent medium surrounded by air, show that the critical angle for total internal reflection and the polarizing angle are related by the expression $\cot \theta_p = \sin \theta_c$.

47. How far above the horizon is the Moon when its image reflected in calm water is completely polarized? ($n_{\text{water}} = 1.33$.)

ADDITIONAL PROBLEMS

48. In Figure P38.42, suppose that the transmission axes of the left and right polarizing disks are perpendicular to each other. Also, let the center disk be rotated on the common axis with an angular speed ω. Show that if unpolarized light is incident on the left disk with an intensity $I_{\max}$, the intensity of the beam emerging from the right disk is

$$I = \frac{1}{16} I_{\max}(1 - \cos 4\omega t)$$

This means that the intensity of the emerging beam is modulated at a rate that is four times the rate of rotation of the center disk. [*Hint:* Use the trigonometric identities $\cos^2\theta = (1 + \cos 2\theta)/2$ and $\sin^2\theta = (1 - \cos 2\theta)/2$, and recall that $\theta = \omega t$.]

49. You want to rotate the plane of polarization of a polarized light beam by 45.0° with a maximum intensity reduction of 10.0%. (a) How many sheets of perfect polarizers do you need to achieve your goal? (b) What is the angle between adjacent polarizers?

50. Figure P38.50 shows a megaphone in use. Construct a theoretical description of how a megaphone works. You may assume that the sound of your voice radiates just through the opening of your mouth. Most of the information in speech is carried not in a signal at the fundamental frequency, but rather in noises and in harmonics, with frequencies of a few thousand hertz. Does your theory allow any prediction that is simple to test?

Figure P38.50 *(Susan Allen Sigmon/Allsport USA)*

51. Light from a helium-neon laser ($\lambda = 632.8$ nm) is incident on a single slit. What is the maximum width for which no diffraction minima are observed?
52. What are the approximate dimensions of the smallest object on Earth that astronauts can resolve by eye when they are orbiting 250 km above the Earth? Assume that $\lambda = 500$ nm and that a pupil's diameter is 5.00 mm.
53. **Review Problem.** A beam of 541-nm light is incident on a diffraction grating that has 400 lines per millimeter. (a) Determine the angle of the second-order ray. (b) If the entire apparatus is immersed in water, what is the new second-order angle of diffraction? (c) Show that the two diffracted rays of parts (a) and (b) are related through the law of refraction.
54. The Very Large Array is a set of 27 radio telescope dishes in Caton and Socorro Counties, New Mexico (Fig. P38.54). The antennas can be moved apart on railroad tracks, and their combined signals give the resolving power of a synthetic aperture 36.0 km in diameter. (a) If the detectors are tuned to a frequency of 1.40 GHz, what is the angular resolution of the VLA? (b) Clouds of hydrogen radiate at this frequency. What must be the separation distance for two clouds at the center of the galaxy, 26 000 lightyears away, if they are to be resolved? (c) As the telescope looks up, a circling hawk looks down. For comparison, find the angular resolution of the hawk's eye. Assume that it is most sensitive to green light having a wavelength of 500 nm and that it has a pupil with a diameter of 12.0 mm. (d) A mouse is on the ground 30.0 m below. By what distance must the mouse's whiskers be separated for the hawk to resolve them?

Figure P38.54 A rancher in New Mexico rides past one of the 27 radio telescopes that make up the Very Large Array (VLA). *© Danny Lehman)*

55. Grote Reber was a pioneer in radio astronomy. He constructed a radio telescope with a 10.0-m diameter receiving dish. What was the telescope's angular resolution for 2.00-m radio waves?
56. A 750-nm light beam hits the flat surface of a certain liquid, and the beam is split into a reflected ray and a refracted ray. If the reflected ray is completely polarized at 36.0°, what is the wavelength of the refracted ray?
57. Light of wavelength 500 nm is incident normally on a diffraction grating. If the third-order maximum of the diffraction pattern is observed at 32.0°, (a) what is the number of rulings per centimeter for the grating? (b) Determine the total number of primary maxima that can be observed in this situation.
58. Light strikes a water surface at the polarizing angle. The part of the beam refracted into the water strikes a submerged glass slab (index of refraction, 1.50), as shown in Figure P38.58. If the light reflected from the upper surface of the slab is completely polarized, what is the angle between the water surface and the glass slab?

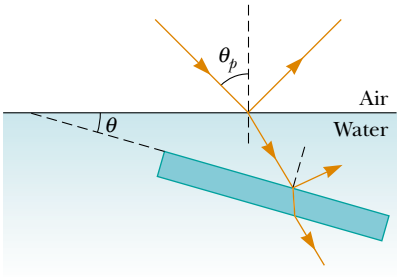


Figure P38.58

59. An American standard television picture is composed of about 485 horizontal lines of varying light intensity. Assume that your ability to resolve the lines is limited only

by the Rayleigh criterion and that the pupils of your eyes are 5.00 mm in diameter. Calculate the ratio of minimum viewing distance to the vertical dimension of the picture such that you will not be able to resolve the lines. Assume that the average wavelength of the light coming from the screen is 550 nm.

60. (a) If light traveling in a medium for which the index of refraction is n_1 is incident at an angle θ on the surface of a medium of index n_2 so that the angle between the reflected and refracted rays is β, show that

$$\tan\theta = \frac{n_2 \sin\beta}{n_1 - n_2 \cos\beta}$$

[*Hint:* Use the identity $\sin(A + B) = \sin A \cos B + \cos A \sin B$.] (b) Show that this expression for $\tan\theta$ reduces to Brewster's law when $\beta = 90°$, $n_1 = 1$, and $n_2 = n$.

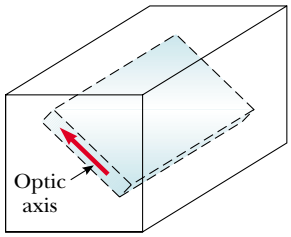
61. Suppose that the single slit in Figure 38.6 is 6.00 cm wide and in front of a microwave source operating at 7.50 GHz. (a) Calculate the angle subtended by the first minimum in the diffraction pattern. (b) What is the relative intensity $I/I_{\max}$ at $\theta = 15.0°$? (c) Consider the case when there are two such sources, separated laterally by 20.0 cm, behind the slit. What must the maximum distance between the plane of the sources and the slit be if the diffraction patterns are to be resolved? (In this case, the approximation $\sin\theta \approx \tan\theta$ is not valid because of the relatively small value of a/λ.)

62. Two polarizing sheets are placed together with their transmission axes crossed so that no light is transmitted. A third sheet is inserted between them with its transmission axis at an angle of 45.0° with respect to each of the other axes. Find the fraction of incident unpolarized light intensity transmitted by the three-sheet combination. (Assume that each polarizing sheet is ideal.)

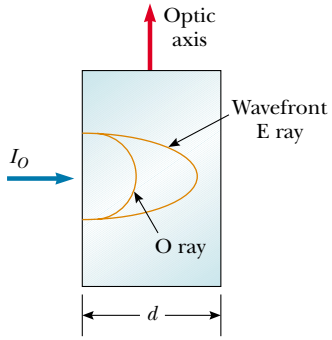
63. Figure P38.63a is a three-dimensional sketch of a birefringent crystal. The dotted lines illustrate how a thin parallel-faced slab of material could be cut from the larger specimen with the optic axis of the crystal parallel to the faces of the plate. A section cut from the crystal in this manner is known as a *retardation plate*. When a beam of light is incident on the plate perpendicular to the direction of the optic axis, as shown in Figure P38.63b, the O ray and the E ray travel along a single straight line but with different speeds. (a) Letting the thickness of the plate be d, show that the phase difference between the O ray and the E ray is

$$\theta = \frac{2\pi d}{\lambda}(n_O - n_E)$$

where λ is the wavelength in air. (b) If in a particular case the incident light has a wavelength of 550 nm, what is the minimum value of d for a quartz plate for which $\theta = \pi/2$? Such a plate is called a *quarter-wave plate*. (Use values of n_O and n_E from Table 38.1.)



(a)



(b)

Figure P38.63

64. Derive Equation 38.12 for the resolving power of a grating, $R = Nm$, where N is the number of lines illuminated and m is the order in the diffraction pattern. Remember that Rayleigh's criterion (see Section 38.3) states that two wavelengths will be resolved when the principal maximum for one falls on the first minimum for the other.

65. Light of wavelength 632.8 nm illuminates a single slit, and a diffraction pattern is formed on a screen 1.00 m from the slit. Using the data in the table on the following page, plot relative intensity versus distance. Choose an appropriate value for the slit width a, and on the same graph used for the experimental data, plot the theoretical expression for the relative intensity

$$\frac{I}{I_{\max}} = \frac{\sin^2(\beta/2)}{(\beta/2)^2}$$

What value of a gives the best fit of theory and experiment?

66. How much diffraction spreading does a light beam undergo? One quantitative answer is the *full width at half maximum* of the central maximum of the Fraunhofer diffraction pattern of a single slit. You can evaluate this angle of spreading in this problem and in the next. (a) In Equation 38.4, define $\beta/2 = \phi$ and show that, at the point where $I = 0.5I_{\max}$, we must have $\sin\phi = \phi/\sqrt{2}$. (b) Let $y_1 = \sin\phi$ and $y_2 = \phi/\sqrt{2}$. Plot y_1 and y_2 on the same set of axes over a range from $\phi = 1$ rad to $\phi = \pi/2$ rad. Determine ϕ from the point of inter-

Relative Intensity	Distance from Center of Central Maximum (mm)
1.00	0
0.95	0.8
0.80	1.6
0.60	2.4
0.39	3.2
0.21	4.0
0.079	4.8
0.014	5.6
0.003	6.5
0.015	7.3
0.036	8.1
0.047	8.9
0.043	9.7
0.029	10.5
0.013	11.3
0.002	12.1
0.000 3	12.9
0.005	13.7
0.012	14.5
0.016	15.3
0.015	16.1
0.010	16.9
0.004 4	17.7
0.000 6	18.5
0.000 3	19.3
0.003	20.2

section of the two curves. (c) Then show that, if the fraction λ/a is not large, the angular full width at half maximum of the central diffraction maximum is $\Delta\theta = 0.886\lambda/a$.

67. Another method to solve the equation $\phi = \sqrt{2}\sin\phi$ in Problem 66 is to use a calculator, guess a first value of ϕ, see if it fits, and continue to update your estimate until the equation balances. How many steps (iterations) does this take?

68. In the diffraction pattern of a single slit, described by the equation

$$I_\theta = I_{\max}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

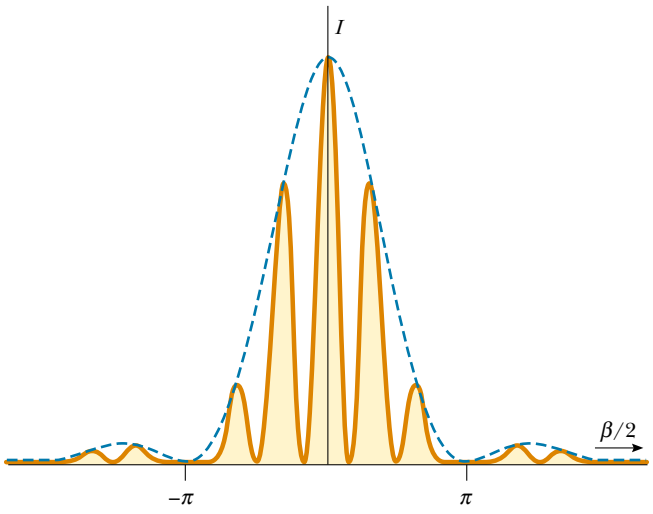
with $\beta = (2\pi a \sin\theta)/\lambda$, the central maximum is at $\beta = 0$ and the side maxima are *approximately* at $\beta/2 = (m + \frac{1}{2})\pi$ for $m = 1, 2, 3, \ldots$. Determine more precisely (a) the location of the first side maximum, where $m = 1$, and (b) the location of the second side maximum. Observe in Figure 38.10a that the graph of intensity versus $\beta/2$ has a horizontal tangent at maxima and also at minima. You will need to solve a transcendental equation.

69. A *pinhole camera* has a small circular aperture of diameter D. Light from distant objects passes through the aperture into an otherwise dark box, falling upon a screen located a distance L away. If D is too large, the display on the screen will be fuzzy because a bright point in the field of view will send light onto a circle of diameter slightly larger than D. On the other hand, if D is too small, diffraction will blur the display on the screen. The screen shows a reasonably sharp image if the diameter of the central disk of the diffraction pattern, specified by Equation 38.9, is equal to D at the screen. (a) Show that for monochromatic light with plane wave fronts and $L \gg D$, the condition for a sharp view is fulfilled if $D^2 = 2.44\,\lambda L$. (b) Find the optimum pinhole diameter if 500-nm light is projected onto a screen 15.0 cm away.

Answers to Quick Quizzes

38.1 The space between the slightly open door and the doorframe acts as a single slit. Sound waves have wavelengths that are approximately the same size as the opening and so are diffracted and spread throughout the room you are in. Because light wavelengths are much smaller than the slit width, they are virtually undiffracted. As a result, you must have a direct line of sight to detect the light waves.

38.2 The situation is like that depicted in Figure 38.11 except that now the slits are only half as far apart. The diffraction pattern is the same, but the interference pattern is stretched out because d is smaller. Because $d/a = 3$, the third interference maximum coincides with the first diffraction minimum. Your sketch should look like the figure to the right.



38.3 Yes, but no diffraction effects are observed because the separation distance between adjacent ribs is so much greater than the wavelength of the x-rays.